THE DYNAMICAL STATE OF THE INTERSTELLAR GAS AND FIELD*

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ABSTRACT

There is now enough observational information available to show that the interstellar magnetic field in the general neighborhood of the Sun is, on the average, parallel to the plane of the Galaxy, with an average strength somewhere between 10^{-6} and 10^{-5} gauss. This paper points out certain dynamical requirements for the existence of such a field. The paper is based on the assumption that the intergalactic medium, whatever it may be, exerts pressures on the Galaxy that are small compared to 10^{-12} dyne/cm^2. It can then be shown that the galactic, or interstellar, magnetic field must be confined to the Galaxy by the weight of the gas threaded by the field and distributed throughout the disk of the Galaxy. It is then shown that the interstellar gas-field system is subject to a universal Rayleigh-Taylor instability of such a nature that the interstellar gas tends to concentrate into pockets suspended in the field. The cause of the instability may be thought of as a hydromagnetic self-attraction in the interstellar gas, which may be ten times larger than the gravitational self-attraction of the gas. It is this hydromagnetic self-attraction which produces the observed tendency of the interstellar gas to be confined in discrete clouds.

The calculations and arguments do not restrict the over-all topology or the strength of the galactic field, which apparently must still be determined from observation.

I. INTRODUCTION

The strength and topology of the galactic magnetic field is a central problem in the origin of cosmic rays, galactic non-thermal radio emission, and the dynamics of the arms of the Galaxy (see Wentzel 1963; Woltjer 1963, 1965; Parker 1966*a*). The early optical polarization measurements of Hiltner (1949, 1951, 1956; Hall 1949) indicate a large-scale average field parallel to the plane of the Galaxy (Davis and Greenstein 1951), but uncertainty in the composition of the interstellar dust grains responsible for the polarization prevents a quantitative estimate of the field strength from these observations (Greenberg 1964). Both the radio observations (Morris and Berge 1964) and the polarization of starlight (Smith 1956; Behr 1959) indicate that the local lines of force lie parallel to the direction of galactic longitude $l^{\text{II}} = 70° \pm 20°$, which agrees with the direction of the spiral arm determined from the distribution of interstellar gas and O associations (Weaver 1953; van de Hulst, Muller, and Oort 1954; Westerhout 1957). Radio observations now seem to place an upper limit of about 5×10^{-6} gauss on the large-scale average field strength in the disk of the Galaxy. Morris and Berge (1964) point out that the Faraday rotation indicates a reversal of the field across the plane of the Galaxy. Thus, taken together, the observations seriously limit the ideas concerning the general nature of the galactic magnetic field. But so far, theory and observation, either separately or together, are unable to give a unique picture of the general galactic (interstellar) field.

It is the purpose of the present paper to point out some theoretical facts that further limit the possible magnetic configurations in the Galaxy. In particular the considerations give an upper limit on the field strength and a unique dynamical structure for the interstellar gas clouds in the galactic magnetic field.

The theoretical facts on which the arguments are based are of an elementary nature, but sometimes a rather tedious calculation is required to establish the individual fact. Hence, in order that the main thread of the argument not be repeatedly interrupted, many of the calculations are placed in the appendices with only a reference to the results of the calculation in the text.

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II. EQUILIBRIUM OF FORCE-FREE FIELDS

It is assumed that, apart from the general rotation of the Galaxy, the galactic magnetic field (*a*) is in a quasi-stationary equilibrium state, (*b*) is limited to the Galaxy and the galactic halo,[1] and (*c*) experiences no significant inward pressure from intergalactic space. With these assumptions as a starting point, the first deduction is that the galactic magnetic field must be confined to the Galaxy by the weight of the gas enmeshed in the field (Biermann and Davis 1960).

It is a simple matter to prove the well-known fact that a magnetic field can be confined only by the weight of the gas through which it penetrates. The appropriate virial equations are worked out in Appendix I.[2] In the classical virial equation the gravitational potential energy must overcome the expansive effects of the kinetic energy $2T$ if a stationary equilibrium is to be achieved. Adding a magnetic field means that the gravitational potential energy must overcome the expansive effects of $2T$ *plus* the total magnetic energy $\int dV\, B^2/8\pi$. Magnetic fields are never self-contained. Their presence increases the tendency for the system to expand, and the expansion effect must be overcome by the weight of the gas distributed along the lines of force. This is all well known (Biermann and Davis 1960).

It is not so widely realized, apparently, that gas clouds from which the magnetic field is excluded do not contribute to confining the galactic field. The physical reason is simply that the galactic field is free to flow around the field-free gas clouds and escape from the Galaxy if not confined by other forces. The formal proof is given in Appendix I. Hence the only means for containing the magnetic field of the Galaxy is the weight of the gas *penetrated* by the field.

The theoretical fact that a magnetic field can be confined to an isolated star system only by the weight of gas threaded by the field permits two distinct possibilities for the galactic magnetic field. The first possibility is that the galactic field in the disk of the Galaxy, where we observe it, is held down pretty much throughout the disk by the weight of the gas there. The second possibility is that the field in the disk is not held down throughout the disk but is confined to the Galaxy by gas in the galactic nucleus. To take the second case first, the field would then be largely force-free throughout the disk of the Galaxy, a possibility that has been considered by a number of authors. In this case, every magnetic line of force must be tied to the galactic nucleus in order to be confined to the Galaxy.[3] From purely geometrical considerations it follows that the field density must increase at least as fast as $1/r^2$ between here and the center of the Galaxy. Indeed, the formal calculation of force-free fields (Lüst and Schlüter 1954; Chandrasekhar 1956) shows that a localized field confined at the origin must increase toward the origin at least as fast as $1/r^3$. Hence a field of 5×10^{-6} gauss at a distance of 10 kpc from the galactic nucleus becomes 5×10^{-3} gauss at a distance of 1 kpc. So strong a field would dominate all interstellar gas motions within 5 kpc of the center of the Galaxy, preventing differential rotation of the gas. The polarization effects and synchrotron radiation from the field toward the center of the Galaxy would be enormous. It is our impression, therefore, that the possibility can be ruled out on the basis of observations.

[1] Sciama (1964) has suggested that the galactic field extends throughout the Local Group. We are skeptical of the idea for the reasons given elsewhere (Parker 1966*a*), but, as a matter of fact, the conclusions of the present paper would not be altered by Sciama's field configuration. The important point is that the field in intergalactic space is assumed to be small compared to the field in interstellar space in both cases.

[2] The virial equation does not take the usual form $2T + \int dV\, B^2/8\pi + \Phi = 0$ (Chandrasekhar and Fermi 1953) because the gravitational potential energy Φ is not entirely the result of *self*-gravitation.

[3] It is not possible to confine lines of force that fail to pass through the nucleus of the Galaxy by linking them through lines that do thread through the nucleus.

Another force-free configuration that has been considered is that the galactic magnetic field is in the form of force-free twisted ropes of magnetic flux extending along the galactic arm. Such a configuration in no way avoids the general virial condition that the field must be confined by the weight of gases, but it is a different situation from that in which all lines are tied straight into the galactic nucleus. The case of a twisted rope is worked out in Appendix II, where it is shown that, unless the external (intergalactic) pressure is equal to half the average internal magnetic pressure, the twisted rope will buckle because of compressive stresses along its length. The system would be so unstable as to transform itself into some other configuration within 10^8 years.[4] The buckling might be stabilized by enough internal gas, of course, but the gas is then confining the field, which is not the possibility under discussion.

Altogether, then, *there is no steady magnetic configuration consistent with present observations that is force-free throughout the disk of the Galaxy and is confined only in the galactic nucleus*. The alternative is to assume that the galactic field in the disk is contained more or less throughout the disk by the weight of the interstellar gas in the disk. The next section works out some of the consequences of this.

III. EQUILIBRIUM OF A FIELD CONFINED TO THE DISK

If we conclude that the galactic field must be contained by the weight of the interstellar gas throughout the disk, the immediate question is what this containment requires in the way of an average interstellar-gas density. Either the tensor virial equations or the hydrostatic pressure equation may be used to treat the problem. To see what is needed, consider the simple case suggested by the polarization observations, that the field in the disk is largely parallel to the disk. Then if the field density is the function $B(z)$ of distance measured perpendicular to the plane of the disk, the condition for quasi-static support of the interstellar gas density $\rho(z)$ against the gravitational acceleration perpendicular to the plane of the Galaxy is

$$\frac{d}{dz}\left(p+P+\frac{B^2}{8\pi}\right)=-\rho(z)\,g(z)\,, \tag{1}$$

where $p(z)$ is the gas pressure and $P(z)$ is the cosmic-ray pressure. In the simplest case, suppose that the three pressures are all proportional, with

$$B^2/8\pi=\alpha p\,,\qquad P=\beta p\,, \tag{2}$$

where α and β are dimensionless constants. The possible variations of α and β with z would not alter the conclusions. Then writing $p=\rho u^2$, where u is the *total* rms random gas velocity in the z-direction, it is readily shown that

$$\rho(z)=\rho(0)\exp\left[-\frac{1}{u^2(1+\alpha+\beta)}\int_0^z dz\,g(z)\right] \tag{3}$$

if u is taken to be independent of z in the first approximation.[5] The density falls by a factor of e in one scale height Λ, where

$$\int_0^{\Lambda} dz\,g(z)=u^2(1+\alpha+\beta)\,. \tag{4}$$

[4] The calculations discussed in §§ IV and V show that it is not possible to stabilize the arm with a dense string of stars along the arm.

[5] The assumption that u is independent of z is equivalent to the assumption that the individual gas-cloud velocities have a Gaussian distribution.

Using the mean-value theorem, we write $\int dz\, g(z)$ as $\Lambda\langle g\rangle_\Lambda$, where $\langle g\rangle_\Lambda$ is the mean value of $g(z)$ in $(0,\Lambda)$. Then

$$\Lambda = \frac{u^2(1+\alpha+\beta)}{\langle g\rangle_\Lambda}. \tag{5}$$

This is the basic equilibrium condition for the simple interstellar gas-field system composed of lines of force more or less parallel to the disk of the Galaxy.

The scale height of the interstellar gas is presently estimated to be 100 pc (Schmidt 1956; van de Hulst 1958; Rougoor 1964). The scale for the distribution of late-type stars, such as K giants is $\lambda \simeq 300$ pc (Oort 1959; Hill 1960), and from their analysis, $\langle g\rangle_\Lambda \simeq 1.3 \times 10^{-9}$ cm/sec^2. The rms velocity of the radio-observed gas clouds in the direction perpendicular to the plane of the Galaxy is 5 km/sec (see discussion in Gould, Gold, and Salpeter 1963).[6] It follows that $1 + \alpha + \beta = 1.5$, although the observational uncertainties are considerable. One might expect that the value 1.5 is an upper limit on $1 + \alpha + \beta$, which gives, then, $\alpha + \beta \lesssim 0.5$.

An upper limit on $\alpha + \beta$ puts a *lower* limit on the interstellar gas density because, from equation (2),

$$\rho = \frac{P + B^2/8\pi}{(\alpha+\beta)u^2}. \tag{6}$$

The cosmic-ray pressure is observed to be about 0.5×10^{-12} dyne/cm^2 (Parker 1966*b*). Suppose B is about 5×10^{-6} gauss. Then $P + B^2/8\pi = 1.5 \times 10^{-12}$ dyne/cm^2. It follows that the lower limit on ρ is 12×10^{-24} gm/cm^3, or about 7 hydrogen atoms/cm^3. Even with $B = 1 \times 10^{-6}$ gauss, the lower limit on the density is 3 atoms/cm^3. Altogether, then, the equilibrium considerations suggest that the interstellar gas density is in excess of the observed atomic hydrogen density of one atom/cm^3.[7] The calculations also make it appear unlikely that the interstellar magnetic-field strength can be much in excess of 5×10^{-6} gauss without enormous interstellar gas densities, of 10 atoms/cm^3 or more, to contain the field.

Now the question that arises is whether the high interstellar gas density can be avoided by some other magnetic configuration or cloud motion. Some authors (Biermann and Davis 1960) have discussed the confinement of the galactic field by the interstellar gas using the scalar virial equation and have achieved an *average equilibrium over three dimensions* with low gas densities by assuming that the gas rotates around the center of the Galaxy a little less rapidly (10–15 km/sec) than the stars. Their conjecture that the gas lags behind the stars may well be correct, but it does not affect the present calculation of equilibrium in the *one direction* perpendicular to the disk of the Galaxy. Differential motions in the plane of the Galaxy do not enter into the equilibrium equation (1).

Twisting the interstellar field into ropes would put some of the tension $B^2/4\pi$ in the field to work confining the field pressure $B^2/8\pi$, thereby assisting the weight of the gas in confining the field. But there is a limit to how much this can do. As shown in Appendix II, only a portion of the magnetic pressure can be confined in this way. Even when $B^2/8\pi$ is dropped completely, the lower limit on the density was estimated at 3 atoms/cm^3.

Altogether, then, the requirement of equilibrium in the direction perpendicular to the disk of the Galaxy would *appear* to demand an interstellar gas density in excess of one

[6] The high-velocity clouds at high galactic latitude (Münch and Zirin 1961; Muller, Berkhuijsen, Brown, and Tinbergen 1963) are not included in this number. Doing so would increase the large gas density calculated from eq. (6).

[7] It has been suggested to us by several people that perhaps the weight of a galactic halo of 10^{-2} atom/cm^3 at 10^6 °K compresses the gas in the disk of the Galaxy to the observed thickness of $2\Lambda = 200$ pc. This is an interesting manifestation of a galactic halo and should be looked into.

atom/cm^3. Other indications and consequences of a high interstellar density have been explored in the published literature from points of view which are different from the approach used here. It is interesting to note them briefly before passing on to the next section and the real point of this paper.

Gould *et al.* (1963) point out that the gravitational acceleration perpendicular to the plane of the Galaxy can be deduced from the observed scale height and velocity of the K giants. The value of g so obtained requires a larger mass in the disk of the Galaxy than the stars plus 1 atom/cm^3. They suggest that the additional mass is interstellar gas with a density of 5 atoms/cm^3, 4 atoms/cm^3 being molecular rather than atomic hydrogen.

Recent work by Toomre (1966) and Julian and Toomre (1966) indicates that an average interstellar density of 3 atoms/cm^3 would make the galactic arms understandable in terms of gravitational forces alone (see also Lin and Shu 1964).

The question of the total interstellar gas density is of central importance in calculating the rate at which cosmic rays are generated in the Galaxy (see discussion in Ginzburg and Syrovatskii 1964; Parker 1966*a*). The calculated rate of generation varies directly with the mean interstellar density and is of the order of 10^{40}–10^{41} ergs/sec with 1 atom/cm^3, approaching the total energy output of all the novae and supernovae in the Galaxy.

It is to be hoped that the interstellar molecular hydrogen can soon be looked for. The importance for further discussion of the galactic gas-field system is obvious. Fortunately the qualitative arguments presented in this paper do not depend on the precise values of either the gas or the field density.

IV. STABILITY OF A FIELD CONFINED TO THE DISK

The arguments presented up to this point have shown that the galactic magnetic field in the disk must be contained by the weight of the gas in the disk. The polarization of starlight suggests that the average field in the disk is parallel to the plane of the disk, and a particularly simple example of such equilibrium was considered in the previous section. The next question concerns the stability of such an equilibrium. The field confined by the weight of the gas is quite different from the laboratory plasma confinement with which we are familiar, where the field confines the gas. So it is necessary to look into the matter rather carefully. We begin with the example employed in the previous section, of a magnetic field of density $B(z)$ in the horizontal y-direction. The gravitational acceleration g is in the negative z-direction and the thermal gas density ρ in the field is supported against gravity by the magnetic field, the thermal gas pressure, and the cosmic-ray gas pressure, i.e., the field and the cosmic rays are confined by the weight of the gas.

Consider first the simple convective interchange of the magnetic lines of force, as sketched in Figure 1. The available information suggests that with parallel magnetic lines of force the system may be weakly unstable. If shearing is present, in which the field is parallel to the plane of the Galaxy but the direction of the field rotates about a vertical axis with increasing height above the plane, or if the field is in twisted ropes, the interchange mode is probably stable. Shearing is a well-known laboratory procedure to eliminate interchange instability in the magnetically confined plasma.

It is more interesting to consider the stability of the system against transverse waves in the magnetic field. This calculation is made in Appendix III for an isothermal atmosphere with the equilibrium pressures of the thermal gas, the magnetic field, and the cosmic-ray gas in the constant ratio $1:\alpha:\beta$ (see eq. [2]). The atmosphere is in a constant gravitational field g in the negative z-direction and self-gravitation is neglected. The pressure and density perturbations of the thermal gas are related by the simple equation of state $\delta p/p = \gamma\delta\rho/\rho$, where γ is a constant. The calculations consider a perturbation with a periodic variation exp iky along the large-scale magnetic field. A requirement that the perturbation vanish at the "base" of the atmosphere, say, at $z = 0$, and remain finite at $z = +\infty$ leads to instability whenever

$$\gamma - 1 < \frac{\alpha/2 + \beta + (\alpha + \beta)^2}{(1 + 3\alpha/2 + \beta)} . \tag{7}$$

The thermal gas by itself would, of course, be stable provided only that $\gamma > 1$. The horizontal magnetic field and the cosmic-ray gas both drive the system toward instability, so that γ must exceed 1 by the amount indicated in expression (7) if the thermal gas is to maintain stability. The equilibrium conditions 7 atoms/cm², $B = 5 \times 10^{-6}$ gauss, and $P = 0.45 \times 10^{-12}$ dyne/cm² give instability for any γ less than 1.35; the conditions 3 atoms/cm³, $B \lesssim 2 \times 10^{-6}$ gauss, and $P = 0.45 \times 10^{-12}$ dyne/cm³ give instability for any $\gamma < 1.36$.

Inelastic cloud collisions and radiative transfer in the interstellar medium are so effective that for perturbations with periods of the order of 10^7–10^8 years, such as we

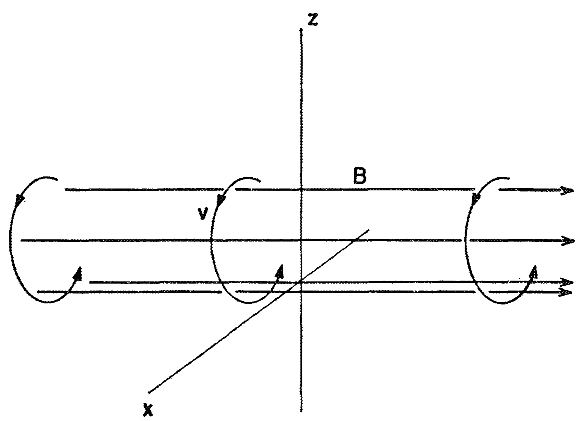

FIG. 1.—Sketch of the convective interchange of parallel magnetic lines of force in the y-direction. The circular velocity of the gas and field is indicated by the velocity symbol v.

are considering here, a density increase in the thermal gas produces little change in the temperature. If there is any effect at all, it is probably for the temperature to decline with increasing density ($\gamma < 1$). So put $\gamma \simeq 1$ as a conservative estimate (see discussion in Parker 1953). The thermal gas is then only marginally stable by itself. *The magnetic field and the cosmic-ray gas make the total gas-field system unstable.*

The calculations show that the growth time is typically 3×10^7 years. This time is short compared to the life of the Galaxy, the time of formation of the galactic arms, and the time in which the thermal gas condenses into stars. So the instability appears to be dynamically important for the state of the interstellar gas-field system. Let us inquire into the nature of the instability. Neither the magnetic field nor the cosmic-ray gas is subject to gravity, so in equilibrium they must be confined by the weight of the thermal gas. That is to say, some of the weight of the thermal gas is supported by the magnetic field and cosmic-ray pressure. So if a perturbation is introduced involving vertical displacement of some portion of the horizontal equilibrium field, the thermal gas tends to slide downward along the magnetic lines of force away from the raised portion of the field into the lower regions along the lines of force. This diminishes the overburden on the raised portion, permitting the field and cosmic-ray gas to expand upward there, causing further slipping of the thermal gas downward along the lines of force, etc. At the same time that the raised portions of the field are being unloaded, the burden on the lower portions is being increased. Only if γ is sufficiently greater than 1 will the thermal gas resist the tendency to slide downward along the field and so give a stable atmosphere.

It is evident that the instability is distinct from the well-known Jeans's gravitational instability, which is the result of self-gravitation. The instability is also distinct from the lack of equilibrium caused by unlimited cosmic-ray inflation of the fields at the surface of the galactic disk (Parker 1965). The instability is related to the familiar Rayleigh-Taylor instability in which a dense fluid supported from beneath by the pressure of a light fluid tends to drip downward through the light fluid.

One may inquire if the instability may be avoided with some magnetic-field configuration other than the simple horizontal field. Consider, for instance, a circular geometry, representing a cross-section of the interstellar gas in a self-gravitating galactic arm. Wrap the magnetic lines of force around the arm so that the tension in the field might stabilize the configuration. The stability of the system is treated in Appendix V. The calculations show that the system is as unstable as the horizontal field. A twisted rope of magnetic field, involving lines of force both along and around the arm, fares no better. The flat and circular geometries considered in the appendices by no means exhaust all the possibilities, of course. For instance, the tidal forces exerted on any one galactic arm by the rest of the Galaxy would distort a circular geometry into an elliptical one. Or the field may be twisted into many small parallel ropes. Or the gravitational field may be taken to be increasing with height. The simple case that g is proportional to z is outlined in Appendix IV. But nothing essentially new is added to the problem by such complications. The basic point is that, if the cosmic rays and/or the lines of force of a large-scale field along the galactic disk or arm are confined by the weight of the thermal gas, then *the gas always tends to drain downward along the magnetic lines of force into the lowest region along each line*. The instability is unavoidable unless the thermal gas is strongly stable by itself. The interstellar thermal gas is not significantly stable by itself ($\gamma \lesssim 1$), so the magnetic field and the cosmic-ray gas drive the interstellar gas-field system unstable in periods of 10^7–10^8 years.

V. THE LONG-TERM STATE OF THE INTERSTELLAR GAS-FIELD SYSTEM

It has been demonstrated that a large-scale equilibrium interstellar magnetic field (suggested by present magnetic observations) is intrinsically unstable in a short time, of only 3×10^7 years. The instability must quickly destroy the equilibrium. The question is, then, what is the dynamical state of the interstellar magnetic field now, after 10^9–10^{10} years? The answer to this question follows from the nature of the instability. (Some examples are worked out in detail in Appendix III.) The instability is the result of the thermal gas draining down along the magnetic lines of force into the low regions along the field, thereby burdening down the low regions and releasing the field between the low regions to expand upward. A sketch of the resulting field configuration along a line of force is shown in Figure 2. The horizontal spacing of the gas pockets in the low regions is of the general order of magnitude of the scale height of the system (see Appendix III). It must be concluded from the calculations that, *if there is a large-scale interstellar field confined to the Galaxy, then the interstellar gas is presently suspended in the field in discrete clouds with separations of the order of* 10–10^3 pc.

It is interesting that the dynamical properties of a large-scale magnetic field should lead to this conclusion, because the conclusion may help resolve a perplexing problem concerning the maintenance of some of the less massive interstellar gas clouds. It is observed (Adams 1949; Münch and Unsöld 1962) that the interstellar gas exists mainly in widely separated discrete clouds (see the recent high-resolution observation of interstellar absorption lines by Livingston and Lynds 1964). The usual explanation for the discrete character of the interstellar gas is self-gravitation of the individual clouds,[8] but there is the problem that in many cases the cloud masses inferred from the observations do not seem to be large enough to maintain the cloud in equilibrium by self-gravitation alone (see Kahn and Dyson 1965). For instance, the self-gravitation of a spherical cloud with a diameter of 20 pc and a density of 10 hydrogen atoms/cm^3 can hold the cloud together only if the internal motions are 0.7 km/sec or less. A higher density of 100 atoms/cm^3 can contain internal motions of only 2.2 km/sec. But even the thermal velocities are this large, to say nothing of the 10-km/sec motions expected from collisions between clouds and from the passage of hot luminous stars through the region. So there is some question as to the means by which the apparent identity of the smaller, more tenuous, interstellar gas clouds is maintained. The new point arising in the galactic field configuration presented in this paper is that the self-gravitation of the individual gas clouds is supplemented, in the configuration shown in Figure 2, by the gravitational field of the Galaxy as a whole.

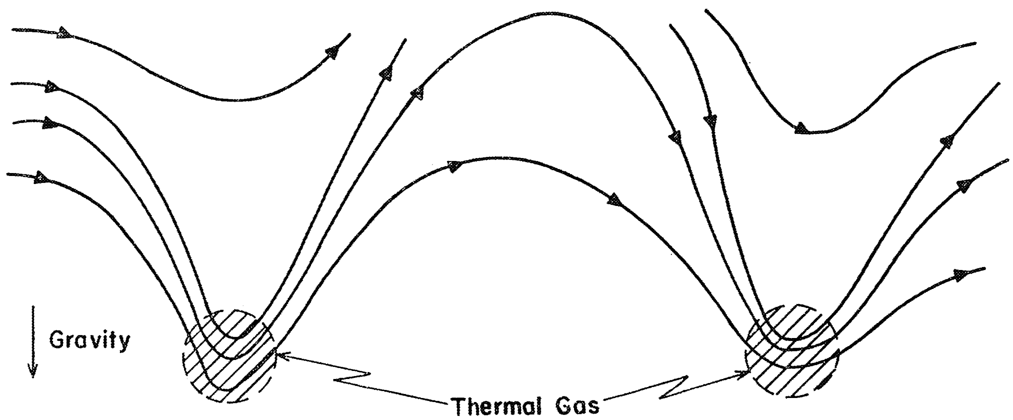

FIG 2.—Sketch of the local state of the lines of force of the interstellar magnetic field and interstellar gas-cloud configuration resulting from the intrinsic instability of a large-scale field along the galactic disk or arm when confined by the weight of the gas.

To illustrate the supplement to self-gravitation in a direct way, and to establish that the supplement may be large in many cases, consider two parallel, widely separated, infinitely long, slender cylinders of gas lying across the horizontal magnetic field B_0 and supported by the magnetic field in the large-scale gravitational field g. The entire region is filled with a tenuous conducting plasma, so that the hydromagnetic equations are the appropriate description of the system. To make the problem tractable, suppose that the pressure of the tenuous plasma is negligible compared to the pressure and weight of the cylinders of dense gas lying across the field. Then $\nabla \times \boldsymbol{B} = 0$, with $\boldsymbol{B} = -\nabla\psi$, $\nabla^2\psi = 0$, everywhere except in the dense cylinders. The field outside the cylinders has the same configuration and stresses as though the space were a vacuum. The solution of this hydromagnetic problem may be effected simply by noting that, if m is the mass per unit length of each cylinder, then the current I induced in each cylinder by the weight on the magnetic field is given by

$$\frac{IB_0}{c} = m\,g\,.$$

[8] Alternatively it has been suggested (Savedoff and Spitzer 1950) that the region between clouds is filled with hot (10^4 ° K) tenuous ($0.1/\text{cm}^3$) hydrogen, whose pressure confines the neutral clouds.

Since the fields outside the cylinders are the same as for two parallel currents I in a vacuum, the fields carry the same stresses as in a vacuum. It is well known that two parallel currents separated by a distance s in a vacuum attract each other with the force

$$F = \frac{2I^2}{c^2 s} = \frac{2}{s} \frac{m^2 g^2}{B_0^2}.$$

The stress in the fields between the two cylinders is the same, provided s is large compared to the diameter of the cylinders, so F is the force of attraction between the two cylinders of gas.[9] Note that this hydromagnetic force of attraction is proportional to the square of the masses and inversely proportional to the distance s, just as the gravitational force

$$F_G = \frac{2Gm^2}{s}.$$

It follows that the ratio of the hydromagnetic force to the gravitation is

$$\frac{F}{F_G} = \frac{g^2}{GB_0^2},$$

which may be extremely large in regions of weak galactic magnetic field B_0 and strong galactic gravitational field g. The simple example of two short segments of length δ supported by the field is easily worked out, showing how the situation is complicated by currents flowing along the magnetic lines of force, in addition to those across.

The gravitational acceleration perpendicular to the disk of the Galaxy is estimated (see, e.g., Oort 1960; Hill 1960; Gould *et al.* 1963) to be of the order of 3×10^{-9} cm/sec^2 at a distance of 100 pc above the central plane of the disk of the Galaxy. A field of $B = 5 \times 10^{-6}$ gauss then gives a ratio $F/F_G = 5$. It is evident from this example that the effective attraction between two elements of gas may be enormously increased above self-gravitation and may, therefore, be an important effect in confining the interstellar gas to discrete clouds.

It is evident that the attraction should be included in calculations of Jeans's instability criterion. To a first approximation the effect may be represented by a suitable increase in the effective gravitational constant G. Hence, for a given average gas density the result is a smaller mass for the individual contracting cells of gas.

It should be noted that the attraction vanishes on the central plane of the disk of the Galaxy because the component g of the gravitational field perpendicular to the plane of the disk vanishes there. It follows, therefore, that there should be some tendency for small tenuous gas clouds (which have little self-gravitation) to be defined more sharply in regions removed from the central plane than in regions near the central plane. It suggests, too, that there may be a tendency for stars of larger mass to be formed near the central plane of the disk.

VI. DISCUSSION

The arguments of the preceding sections have passed rather directly to the conclusions, leaving a number of important points untouched. This section is intended to go back and pick up some of these points.

First of all, it should be noted that the calculations and conclusions presented in this paper do not restrict the over-all magnetic configuration of the galactic magnetic field. The calculations have to do only with the small-scale (10–10^3 pc) properties of the

[9] The interested reader may wish to carry through the exercise of computing F formally from the hydromagnetic equations.

galactic field. They apply to a twisted rope of magnetic flux along a galactic arm as well as to a large-scale horizontal field in the disk, etc.

The arguments began with the idea of a large-scale field with a tendency to lie parallel to the disk of the Galaxy, because the observed polarization of the light of distant stars seems to require this. The question is, then, whether the final pendulant configuration (Fig. 2) is consistent with this starting point of view. We suggest that it is. The space average of the vertical component of the magnetic field is zero in a gas cloud and in the intercloud region as well. The light path for the significantly reddened stars, in which the polarization is observed, usually passes through more than one gas cloud (see again Livingston and Lynds 1964). Presumably, therefore, the net direction of polarization of the light of most reddened stars comes close to the over-all average field in the disk, which the observations (Hiltner 1949, 1951, 1956) show (Davis and Greenstein 1951) is close to the plane of the Galaxy. It must be remembered that the averaging is generally believed to be sufficient to obscure even a possible over-all twisting of magnetic lines of force along the galactic arm.

The velocities of the individual interstellar gas clouds relative to the local galactic rotation are statistically isotropic so far as observations can tell. And, so far as observations can tell, the magnitude of the random velocities does not vary with height above the plane of the Galaxy. The interstellar cloud system sketched in Figure 2 seems to fit this picture fairly well. If the motion of each cloud can be considered a harmonic oscillation about some equilibrium position, then random excitation of the oscillator leads to a statistically isotropic velocity no matter how weak the binding may be in one direction and how strong in another.[10] Since the gas clouds are all tied into the same large-scale magnetic-field system, it is not surprising if the degree of excitation is independent of distance from the galactic plane.

VII. SUMMARY AND CONCLUSIONS

The main argument presented in this paper is based on two premises: (*a*) There exists a large-scale magnetic field in the Galaxy; (*b*) the magnetic field, and the cosmic rays trapped in it, are not confined to the Galaxy by extragalactic pressures. A large-scale field of 10^{-6}–10^{-5} gauss is suggested by the observed polarization of starlight, by Faraday rotation, and by the apparently steady nature of the cosmic-ray intensity. It was then argued that the field and cosmic rays must be confined throughout the disk of the Galaxy by the weight of the interstellar gas threaded by the field. But the effective γ for slow changes of density of the interstellar gas is apparently not enough greater than 1 to provide stable confinement of the magnetic field and/or the cosmic rays. The calculations show that the result is an instability, in the form of a strong tendency for the interstellar gas to clump together into discrete clouds. The clumping tendency can be represented by a pseudo-self-gravitation of the interstellar gas which is larger by a factor of the order of g^2/GB^2 than the true self-gravitation. The enhanced clumping explains the coherent nature of many of the interstellar clouds whose masses are otherwise too small to hold them together.

The success of the general dynamical picture in explaining the discrete nature of many interstellar clouds suggests that the dynamical considerations presented here are a major effect in determining the state of the interstellar gas and fields. The clumping must be included in the treatment of star formation in interstellar gas clouds.

The author wishes to express his gratitude to Dr. Ian Lerche for stimulating discussion of the dynamical properties of the interstellar medium.

[10] The *displacement* amplitudes are not isotropic in an anisotropic oscillator.

APPENDIX I

THE VIRIAL EQUATIONS FOR THE GAS-FIELD SYSTEM

The arguments presented in § III are based on the result, easily shown from the virial equations, that a galactic magnetic field can be confined to the Galaxy only by the weight of the gas which it penetrates.

Consider the virial equation for the interstellar gas and magnetic-field system. Let ρ represent the gas density and B_i the field. Let ϕ represent the total gravitational potential, due to stars and gas together. Then (Chandrasekhar and Fermi 1953; Parker 1954)

$$\frac{d^2 I_{ij}}{dt^2} = 4T_{ij} + \int dS_k (x_i M_{jk} + x_j M_{ik}) - 2\int dV M_{ij} - \int dV \rho \left(x_j \frac{\partial \phi}{\partial x_i} + x_i \frac{\partial \phi}{\partial x_j} \right), \quad \text{(I.1)}$$

where I_{ij} is the moment-of-inertia tensor and T_{ij} is the kinetic tensor:

$$I_{ij} = \int dV \rho x_i x_j , \qquad T_{ij} = \tfrac{1}{2} \int dV \rho \frac{dx_i}{dt} \frac{dx_j}{dt} . \quad \text{(I.2)}$$

It will be assumed that there are no external forces exerted on the gas-field system, so the surface integral vanishes when calculated over a sufficiently distant external inclosing surface.

If the gravitational potential ϕ were the result only of the gas density ρ ($\nabla^2 \phi = 4\pi G \rho$), it would be possible to write the last term on the right-hand side of expression (I.1) in terms of the total gravitational potential $\Phi = \int dV \rho \phi$ in the usual way. But since other matter, such as the stars, contributes to ϕ, this is not possible. The precise value of the integral depends now on the spatial form of ϕ. To pick a simple example, suppose that the gas-field system is in a spherically symmetric parabolic potential well,

$$\phi(r) = g \frac{r^2}{2a} , \quad \text{(I.3)}$$

where g is the inward gravitational acceleration at the characteristic radial distance $r = a$. Then it follows that

$$\int dV \rho \left(x_i \frac{\partial \phi}{\partial x_j} + x_j \frac{\partial \phi}{\partial x_i} \right) = \frac{2g}{a} \int dV \rho x_i x_j = \frac{2g}{a} I_{ij} , \quad \text{(I.4)}$$

and the trace of this is

$$\frac{2g}{a} I_{ii} = \int dV \rho 2 x_i \frac{\partial \phi}{\partial x_i} = 4 \int dV \rho \phi \equiv 4\Phi . \quad \text{(I.5)}$$

Thus, if there are no forces on the external surface of the system, equation (I.1) reduces to

$$\frac{d^2 I_{ij}}{dt^2} + \frac{2g}{a} I_{ij} = 4T_{ij} - 2\int dV M_{ij} . \quad \text{(I.6)}$$

Contracting on the indices gives

$$2\Phi = 2T + \int dV B^2/8\pi \quad \text{(I.7)}$$

for equilibrium, where T is the total kinetic energy of the internal motions and $\int dV\, B^2/8\pi$ is recognizable as the total magnetic energy of the system. The gravitational potential energy Φ is measured above the potential at the origin, rather than below the potential at infinity. Equation (I.7) illustrates the fact that the kinetic energy and the magnetic energy must be contained by a comparable amount of gravitational potential energy. A magnetic field is not self-containing. It must be confined by the weight of the gas which it threads.

There has been some discussion of the possibility of interstellar gas clouds from which the galactic magnetic field is excluded for one reason or another (see Woltjer 1963). The diamagnetic clouds lie outside the gas-field system with which we are concerned, but they exert forces on the gas-field system at their surfaces. It is necessary to work out the surface integrals in the contracted form in (I.1), yielding $\int dS_j x_i M_{ij} = \int dS_i x_i B^2/8\pi$ from the fact that $dS_i B_i = 0$ for clouds which exclude the external field. To make the problem tractable, suppose that the diagmagnetic clouds are spherical with radius R, widely separated, and small compared to the scale L of the external field. Consider a cloud with its center at $x_i = X_i$. Let the field in that neighborhood be $e_i B(X_j)$ before the cloud was introduced, where e_i is the unit vector. Following the introduction of the cloud, the field in that neighborhood becomes $-\partial\psi/\partial x_i$ with

$$\psi = B(X_i)\left[\left(1+\frac{R^3}{2r^3}\right)\xi_i e_i + O\left(\frac{R}{L}\right)\right], \tag{I.8}$$

where ξ_i represents rectangular coordinates measured from the center of the sphere and r is the radial distance $(\xi_i\xi_i)^{1/2}$ from the center. It is readily shown that the work W required to inflate the sphere to a radius R against the pressure exerted on it by the external field is

$$W = \frac{3}{2}\left(\frac{4\pi R^3}{3}\right)\frac{B^2(X_i)}{8\pi} = \frac{1}{4}B^2(X_j)R^3 . \tag{I.9}$$

The external field $B(X_j)$ is not uniform, of course, so there will be a net buoyant force (Parker 1955, 1957) exerted on the cloud. The force F_i is

$$F_i = -\frac{\partial W}{\partial x_i}, \tag{I.10}$$

where the differentiation is carried out with R fixed.

To evaluate the surface integral write $x_i = X_i + \xi_i$. Then, for each diamagnetic cloud,

$$\int dS_i x_i \frac{B^2}{8\pi} = X_i \int dS_i \frac{B^2}{8\pi} + \int dS_i \xi_i \frac{B^2}{8\pi} .$$

The first integral on the right-hand side is just the negative of the total force exerted on the cloud by the external magnetic field. The second integral is easily shown to be equal to $3W$, since $dS_i\xi_i = 2\pi R^3 \sin\theta d\theta$ and $B^2 = 9B^2(X_j)\sin^2\theta/4$, where $\cos\theta = e_i\xi_i/r$. Hence

$$\int dS_i x_i \frac{B^2}{8\pi} = -X_i F_i + 3W = X_i \frac{\partial W}{\partial X_i} + 3W = \frac{\partial}{\partial X_i}(W X_i) . \tag{I.11}$$

It follows that the scalar virial equation for the gas-field system external to a number of diagmagnetic spheres is

$$2\Phi = 2T + \int dV \frac{B^2}{8\pi} + \sum \frac{\partial}{\partial X_i} W X_i \tag{I.12}$$

in place of equation (I.7). The sum is over all the spheres. The sum can be greatly simplified if we consider a very large number of small diamagnetic spheres distributed over the cloud. If $\Upsilon(x_i)$ is the number of clouds per unit volume, then Σ can be replaced by $\int dV \Upsilon$ and

$$\sum \frac{\partial}{\partial X_i} W X_i = \int dV \Upsilon \frac{\partial}{\partial x_i}(W x_i) .$$

The total contribution of the diamagnetic spheres to the right-hand side of equation (I.12) is thus

$$S = \int dV \Upsilon \left[\frac{\partial}{\partial x_i}(W x_i) + W \right],$$

where the first term in the integrand represents the contribution of the surface integral and the second term represents the increased magnetic energy. The integration is over the entire volume occupied by the gas-field system and it is assumed, therefore, that the diamagnetic cloud density Υ vanishes at the surface of the system. Integrating by parts, then, leads to

$$S = \int dV \Upsilon W \left(1 - \frac{x_i}{\Upsilon} \frac{\partial \Upsilon}{\partial x_i} \right).$$

Both Υ and W are positive or zero everywhere throughout the volume. So the sign of S is determined by the quantity in parentheses. The gravitational field of the Galaxy causes Υ to decline outward from the center of the system so that $x_i \partial \Upsilon / \partial x_i$ is generally negative, except for local fluctuations. But the quantity in parentheses is always positive provided that $(x_i/\Upsilon) \partial \Upsilon / \partial x_i < + 1$. Hence we conclude that $S > 0$ for the real situation encountered in the Galaxy. The contribution of diamagnetic gas clouds is to increase the tendency for the system to expand. Gas clouds from which the magnetic field is excluded do not confine the magnetic field to the Galaxy.

APPENDIX II

FORCES IN A TWISTED ROPE OF MAGNETIC FIELD

A few remarks are required concerning the stresses in a force-free twisted rope of flux, sometimes considered as a possible configuration for the galactic-arm field. The magnetic field in an axially symmetric force-free tube of flux can be expressed (Schlüter, Trefftz, and Lüst 1953) in terms of a generating function $F(\varpi)$, where ϖ is distance measured from the axis of the tube, along which distance is measured by z and around which the azimuthal angle is ϕ. The components of the magnetic field can always be written as

$$B_\phi^2 = -\tfrac{1}{2}\varpi \frac{dF}{d\varpi}, \qquad B_z^2 = F(\varpi) + \tfrac{1}{2}\varpi \frac{dF}{d\varpi},$$

so that the arbitrary generating function is just the square of the magnitude of the field. The total tension Q in the rope, from $\varpi = 0$ out to $\varpi = a$, is

$$\begin{aligned} Q(a) &= 2\pi \int_0^a d\varpi\, \varpi \left(\frac{B_z^2 - B_\phi^2}{8\pi} \right) = \tfrac{1}{4} \int_0^a d\varpi\, \varpi \frac{d}{d\varpi}(\varpi F) \\ &= 2\pi a^2 \left[\frac{B^2(a)}{8\pi} - \frac{1}{2\pi a^2} \left(2\pi \int_0^a d\varpi\, \varpi \frac{B^2}{8\pi} \right) \right] \end{aligned} \qquad \text{(II.1)}$$

after integration by parts. The tube of force can be stable only if $Q > 0$, for if $Q < 0$ the tube is under compression and will buckle. Suppose that the rope terminates at some finite radius $\varpi = a$. There must then be an external pressure equal to $B^2(a)/8\pi$ exerted on the field at $\varpi = a$, of course. Then it may be seen from (II.1) that $Q(a) > 0$ for stability requires that the external pressure must equal or exceed one-half the average magnetic energy density within $\varpi = a$. So any such force-free tube must be contained by an external pressure comparable to $B^2/8\pi$ within the tube. This is, of course, nothing more than the virial condition again, stating that a magnetic field will expand to infinity unless confined by comparable inward forces.

APPENDIX III

STABILITY AGAINST TRANSVERSE WAVES

Consider the stability of a composite atmosphere of thermal gas, magnetic field $\boldsymbol{e}_y B(z)$, and cosmic-ray gas against transverse waves propagating along the magnetic field. For this first calculation suppose that the gravitational acceleration g and the thermal gas temperature T are independent of x,y,z. It will be found convenient to introduce the thermal velocity $u = (kT/M)^{1/2}$. Then the pressure and density are related by $p(z) = u^2\rho(z)$. It will be assumed that the cosmic-ray gas maintains its statistical isotropy while flowing along the magnetic lines of force in the slow (10^7 years) distortions of the field to be considered here. Suppose that the magnetic and cosmic-ray pressures are confined by the weight of the thermal gas and are simply proportional to the thermal gas pressure at each point, so that equation (2) in the text may be applied. Hydrostatic equilibrium leads to

$$\frac{1}{p}\frac{dp}{dz} = -\frac{g}{u^2(1+\alpha+\beta)} = \frac{1}{\rho}\frac{d\rho}{dz} = \frac{1}{P}\frac{dP}{dz} = \frac{2}{B}\frac{dB}{dz} \equiv -\frac{1}{L}, \tag{III.1}$$

where L is the scale height of the atmosphere. Introduce a perturbation $\exp i\omega t$ with wave vector parallel to the yz-plane[11] involving the velocity components v_y and v_z. Express the magnetic perturbation associated with v_y and v_z as the curl of the vector potential $\boldsymbol{e}_x \delta A(y,z)$, where $\boldsymbol{e}_x$ represents a unit vector in the x-direction. The hydromagnetic equation for δA becomes

$$\frac{\partial \delta A}{\partial t} = -v_z B(z). \tag{III.2}$$

If the thermal gas density and pressure perturbations are $\delta\rho$ and δp, and if the cosmic-ray gas pressure perturbation is δP, then the linearized equations of motion[12] are

$$\rho\frac{\partial v_y}{\partial t} = -\frac{\partial \delta p}{\partial y} - \frac{\partial \delta P}{\partial y} - \frac{1}{4\pi}\frac{dB}{dz}\frac{\partial \delta A}{\partial y}, \tag{III.3}$$

$$\rho\frac{\partial v_z}{\partial t} = -\frac{\partial \delta p}{\partial z} - \frac{\partial \delta P}{\partial z} - \frac{B}{4\pi}\nabla^2 \delta A - \frac{1}{4\pi}\frac{dB}{dz}\frac{\partial \delta A}{\partial z} - g\,\delta\rho, \tag{III.4}$$

$$\frac{\partial \delta\rho}{\partial t} + v_z\frac{d\rho}{dz} + \rho\left(\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0, \tag{III.5}$$

$$\frac{\partial \delta p}{\partial t} + v_z\frac{dp}{dz} + \gamma u^2\rho\left(\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0, \tag{III.6}$$

assuming that the pressure variation δp in a given element of gas varies as

$$\frac{\delta p}{p} = \gamma\frac{\delta\rho}{\rho},$$

where γ is a constant.

[11] This excludes the interchange mode, which involves a wave vector principally in the xy-plane.

[12] These hydrodynamic equations neglect particle-wave resonances (Tidman 1966) which are small in the present case because the wavelengths are of the order of 10^{20} cm, traversed in 3×10^9 sec by relativistic particles. Resonance with the cyclotron motion of the cosmic-ray protons around a galactic field of 5×10^{-6} gauss occurs only with protons with energies of 5×10^7 times the proton rest energy. Such particles constitute less than 10^{-11} of all cosmic rays, and it can be shown from Tidman's calculations that the effects are negligible.

The cosmic-ray pressure perturbation is computed from the fact that the cosmic-ray gas is not significantly affected by gravity and the speed of sound in the cosmic-ray gas is very much larger than any of the wave velocities to be considered here. Hence the cosmic-ray gas pressure is uniform along any given line of force. The volume of a tube of force does not vary to first order in the perturbation, so to the order considered, $d\delta P/dt = 0$, i.e.,

$$\frac{\partial \delta P}{\partial t} + v_z \frac{dP}{dz} = 0 . \tag{III.7}$$

To solve these equations solve expression (III.2) for v_z and substitute into equation (III.7). Integrating the result over time gives

$$\delta P = \frac{\beta u^2}{B} \frac{d\rho}{dz} \delta A . \tag{III.8}$$

Differentiate equation (III.3) with respect to time and use equations (III.6) and (III.8) to eliminate δp and δP. The result may be written

$$Q^2 v_y = -\frac{u^2}{B}\left[\frac{1}{\rho}\frac{d\rho}{dz}\left(1+\alpha+\beta-\frac{\gamma}{2}\right)+\gamma\frac{\partial}{\partial z}\right]\frac{\partial^2 \delta A}{\partial t \partial y}, \tag{III.9}$$

where Q^2 is the acoustic wave operator,

$$Q^2 \equiv \frac{\partial^2}{\partial t^2} - \gamma u^2 \frac{\partial^2}{\partial y^2}.$$

It is now possible to return to the equations (III.5) and (III.6) for $\delta\rho$ and δp and employ expression (III.2) to eliminate v_z with equation (III.9) to eliminate v_y. The results can be integrated once over time yielding

$$Q^2 \frac{\delta\rho}{\rho} = \frac{1}{B}\left\{\left(\frac{1}{2\rho}\frac{d\rho}{dz}+\frac{\partial}{\partial z}\right)Q^2 \delta A + u^2\left[\left(1+\alpha+\beta-\frac{\gamma}{2}\right)\frac{1}{\rho}\frac{d\rho}{dz}+\gamma\frac{\partial}{\partial z}\right]\frac{\partial^2 \delta A}{\partial y^2}\right\}, \tag{III.10}$$

$$\begin{aligned} Q^2 \frac{\delta p}{p} = \frac{1}{B}\Bigg\{&\left[\frac{(1-\gamma/2)}{\rho}\frac{d\rho}{dz}+\gamma\frac{\partial}{\partial z}\right]Q^2 \delta A \\ &+\gamma u^2\left[\left(1+\alpha+\beta-\frac{\gamma}{2}\right)\frac{1}{\rho}\frac{d\rho}{dz}+\gamma\frac{\partial}{\partial z}\right]\frac{\partial^2 \delta A}{\partial y^2}\Bigg\}. \end{aligned} \tag{III.11}$$

The final step is to operate on equation (III.4) with Q^2 and eliminate v_z with the aid of equation (III.2), and $\delta\rho$ and δp with the aid of equations (III.10) and (III.11). The result can be written

$$\begin{aligned} u^2\left[(\gamma+2\alpha)Q^2+\gamma^2 u^2\frac{\partial^2}{\partial y^2}\right]\frac{\partial^2 \delta A}{\partial z^2}+\Bigg\{u^2\left[2\alpha\frac{\partial^2}{\partial y^2}-\frac{g^2}{2u^4}\frac{(\alpha+\gamma/2)}{(1+\alpha+\beta)^2}\right]Q^2 \\ -g^2\frac{(1+\alpha+\beta-\gamma/2)^2}{(1+\alpha+\beta)^2}\frac{\partial^2}{\partial y^2}\Bigg\}\delta A - Q^2\frac{\partial^2 \delta A}{\partial t^2} = 0 \end{aligned} \tag{III.12}$$

after collecting terms and using the equilibrium conditions to eliminate $d\rho/dz$.

Now suppose that δA is of the form

$$\delta A = f(\xi) \exp(i\omega t + iky) , \tag{III.13}$$

where $\xi = kz$. The equation reduces to

$$\left[2\alpha\gamma - \frac{\omega^2}{k^2u^2}(\gamma + 2\alpha)\right]\frac{d^2 f}{d\xi^2} + \left[\frac{(1+\alpha+\beta-\gamma)(1+\alpha+\beta) - \alpha\gamma/2}{k^2L^2} - 2\alpha\gamma + \frac{\omega^2}{k^2u^2}(2\alpha+\gamma)\left(1+\frac{1}{4k^2L^2}\right) - \left(\frac{\omega^2}{k^2u^2}\right)^2\right] f = 0 \,, \qquad \text{(III.14)}$$

where L is the scale height $g/u^2(1 + \alpha + \beta)$. We impose the boundary condition that the perturbation δA vanish at the base of the atmosphere $z = 0$ and remain finite as $z \to +\infty$. In order to obtain solutions which satisfy both conditions it is necessary that the coefficients of f and $d^2f/d\xi^2$ have the same sign. Hence unstable solutions (k real and $\omega = -i/\tau$ where $\tau > 0$) occur provided only that

$$\frac{(1+\alpha+\beta-\gamma)(1+\alpha+\beta)-\alpha\gamma/2}{k^2L^2} > 2\alpha\gamma + U^2(2\alpha+\gamma)\left(1+\frac{1}{4k^2L^2}\right) + U^4 \,, \qquad \text{(III.15)}$$

where $U \equiv 1/ku\tau$. For marginal stability ($U = 0$) there are solutions provided only that

$$Y > Z\alpha\gamma k^2L^2 \,, \qquad \text{(III.16)}$$

where $Y \equiv (1 + \alpha + \beta)(1 + \alpha + \beta - \gamma) - \alpha\gamma/2$. Thus instability occurs first at long wavelengths $kL \to 0$, and requires only that $Y > 0$. The result is simply interpreted. If $\gamma > 1$ in the absence of magnetic field and cosmic rays ($\alpha = \beta = 0$), there are no solutions with marginal instability. The atmosphere is stable. The effect of the field and/or the cosmic rays is instability, so that if α and β are sufficiently large, they both drive the system to instability no matter how stable the thermal gas might be itself. The effective γ for the interstellar thermal gas is 1 or less,[13] so that there is instability at long wavelengths for any $\alpha,\beta > 0$.

The next question is whether the instability grows sufficiently rapidly to be a significant effect. To demonstrate the growth rate, write $x = 1/k^2L^2$, $n = L/u\tau$, and $U = nx^{1/2}$, where n specifies the growth rate in terms of the time required to move one scale height at a speed u. Typically $L = 100$ pc, $u = 10$ km/sec, so that if $\tau \lesssim 10^8$ years, we must have $n \gtrsim 0.1$. The condition (III.15) for instability becomes $y(x) < 0$, where

$$y(x) \equiv x^2[n^4 + n^2(2\alpha + \gamma)/4] + 2\alpha\gamma + x[n^2(2\alpha + \gamma) + \alpha\gamma/2 - (1 + \alpha + \beta - \gamma)(1 + \alpha + \beta)] \,. \qquad \text{(III.17)}$$

It is obvious that $y(x)$ becomes large without limit as $x \to \pm\infty$.[14] There is a minimum value of y at $x = 2b/a$ where $b = Y - n^2(2\alpha + \gamma)$, $a = n^2(4n^2 + 2\alpha + \gamma)$. The minimum value of y is negative, giving instability, provided only that

$$b^2 > 2\alpha\gamma a \,. \qquad \text{(III.18)}$$

The condition for marginal stability was $Y > 0$. For n as small as 0.1, corresponding to a growth period of 10^8 years, a is of the order of 10^{-2} and $b \simeq Y$ so that expression (III.18) is hardly more than the requirement for marginal stability. The interstellar particle-field system is always unstable under these circumstances because of the apparently low value of γ and the fact that $\alpha,\beta > 0$.

[13] The slow compression of the interstellar gas involved here (10^7–10^8 years) tends to lower the temperature in most cases (Parker 1953) by virtue of the greater ability to radiate (Savedoff and Spitzer 1950).

[14] Negative values of x are physically uninteresting because k is imaginary then. Hence we require that $b > 0$.

To calculate the maximum growth rate n, note that when $(\gamma - 2\alpha)Y \ll (\gamma + 2\alpha)(\alpha\gamma + Y)$ the maximum value of n for which expression (III.18) is satisfied and $b > 0$ is

$$n_{\max} \cong \frac{Y}{[2(\gamma+2\alpha)(\alpha\gamma+Y)]^{1/2}}.$$

This maximum n is associated with infinite wavelength in the vertical direction, so it should not be taken too seriously. The effective maximum n is somewhat less. For a galactic magnetic field of 5×10^{-6} gauss, a cosmic-ray pressure of 0.45×10^{-12} dyne/cm^3, and a thermal gas density of 3 hydrogen atoms/cm^3 with an rms velocity $u = 8$ km/sec, it follows that $\rho u^2 = 3.0 \times 10^{-12}$ dyne/cm^3, $\alpha = 0.3$, and $\beta = 0.15$. Put $\gamma = 1$. Then $Y = 0.50$ and the maximum growth rate is $n = 0.3$, giving $\tau \simeq 3 \times 10^7$ years. A much weaker galactic field permits equilibrium with 1 atom/cm^3, giving $\rho u^2 = 1 \times 10^{-12}$ dyne/cm^3 with $\alpha \simeq 0$ and $\beta = 0.45$. With $\alpha \simeq 0$ the requirement (III.18) becomes $Y > n^2$ for instability, and hence the maximum growth rate is $n_{\max} = 0.807$, giving $\tau = 1.2 \times 10^7$ years.

It is evident that a large number of cases could be investigated, involving various values of α,β,γ and employing different boundary conditions, including cutting off the atmosphere at some specified height $z = h$ with a uniform cosmic-ray gas pressure P_0 or a large-scale uniform magnetic field B_0 beyond. The reader who wishes to investigate the possible effects of significant intergalactic pressure may find some interest in this. Stability can be achieved under some circumstances, as in the limit of strong intergalactic magnetic fields. The stability arises from the fact that with an external pressure the weight of the gas no longer is responsible for confining the magnetic field and cosmic rays. It is the weight of the gas on the field which produces the instability.

We shall content ourselves here with two simple examples of the nature of the unstable flow to illustrate the draining of the thermal gas into the low regions along the lines of force. Suppose that the cosmic-ray gas is absent and the thermal gas is cold. Then $\beta = 0$, $u^2 = 0$, $\alpha u^2 = \frac{1}{2}V_A{}^2 \neq 0$, where V_A is the Alfvén speed $B/(4\pi\rho)^{1/2}$. Equation (III.14) reduces to

$$\frac{d^2 f}{dz^2} + K^2 f = 0,$$

where, with $s^2 \equiv V_A{}^2\tau^2/L^2$,

$$K^2 = \tfrac{1}{4}k^2 s^2\left[1 - \frac{4}{s^2}\left(1 + \frac{4}{k^2L^2}\right) - \left(\frac{2}{s^2k^2L^2}\right)^2\right] \cong \tfrac{1}{4}k^2 s^2$$

in the limit of long growth times, $s^2 \gg 1$. In order that δA vanish at $z = 0$, put

$$\delta A = \epsilon BL \exp\frac{t}{\tau}\cos ky \sin Kz,$$

where $\epsilon \ll 1$. It follows that the magnetic lines of force are given by

$$z - z_0 = \epsilon L \exp\frac{t}{\tau}\sin Kz_0(1 - \cos ky),$$

where z_0 is the value of z at which the line crosses the z-axis. It is readily shown from expression (III.9) that

$$v_y = -\frac{\epsilon}{2}V_A skL \exp\frac{t}{\tau}\sin ky \sin Kz$$

and from expression (III.2) that

$$v_z = -\epsilon\frac{V_A}{s}\exp\frac{t}{\tau}\cos ky \sin Kz.$$

It is evident that the motion drains the thermal gas away from the high regions $ky = \pm(2n + 1)\pi$ along the magnetic lines of force into the low regions $ky = \pm 2n\pi$, as illustrated in Figure 2. When $s \gg 1$, the motion is principally in the horizontal direction.

Suppose, on the other hand, that the magnetic field is very weak,[15] the thermal gas is cold, and most of the pressure is contributed by the cosmic-ray gas. Then $\alpha \simeq 0$, $u^2 = 0$, $\beta u^2 \equiv C^2 \neq 0$, where C is the equivalent thermal velocity, $C = (P/\rho)^{1/2}$. The atmosphere is unstable for any value of $C > 0$. Equation (III.14) reduces to

$$\beta^2\left[\frac{1}{k^2L^2} - \left(\frac{1}{k^2C^2\tau^2}\right)^2\right]f = 0 .$$

Thus, the non-trivial unstable solution is $k^2C^2\tau^2 = kL$ with f any arbitrary function of z, provided only that f is single valued and continuous so as not to violate the assumptions which went into the initial linearization of the equations. The reason for the arbitrariness of f is that neither the thermal gas nor the vanishing magnetic field can resist compression, and the cosmic-ray gas avoids compression by redistributing itself along the magnetic lines of force.

Put

$$\delta A = \epsilon L \exp\frac{t}{\tau} \cos ky f(z) ,$$

and suppose that $df/dz = O(kf)$ for all values of z. Then the magnetic lines of force are given by

$$z - z_0 = \epsilon L \exp\frac{t}{\tau}(1 - \cos ky) f(z_0) ,$$

and

$$v_y = -\epsilon\frac{L}{\tau}\exp\frac{t}{\tau}\sin ky f(z) , \qquad v_z = -\epsilon\frac{L}{\tau}\exp\frac{t}{\tau}\cos ky f(z) .$$

Thus again the motion represents a draining of the thermal gas along the magnetic lines of force from the high regions into the low regions.

APPENDIX IV

VARIATION OF THE GRAVITATIONAL FIELD

The calculations in Appendix III were carried out for the simple case that the gravitational acceleration g is independent of height z. It is physically obvious that the instability does not depend on this simplification for its existence, but it is not without interest to examine what quantitative changes may be expected when the actual variation of g with height above the central plane of the Galaxy is taken into account.

For the case that the gas is cold, $u^2 = 0$, and there is no cosmic-ray pressure, write $V_A^2 = 2\alpha u^2$, where V_A is the Alfvén speed that is independent of height. It is readily shown that in place of equation (III.12) the equation for δA is

$$\frac{\partial^4 \delta A}{\partial t^4} + \left[\frac{g^2(z)}{V_A^2} - V_A^2\nabla^2\right]\frac{\partial^2 \delta A}{\partial t^2} + g^2(z)\frac{\partial^2 \delta A}{\partial y^2} = 0 .$$

As a first approximation write $g(z) = g_0kz$, where g_0k is a constant. Then the equation has solutions of the form

$$\delta A = \exp\left(\frac{t}{\tau} + iky + i\frac{\xi^2}{2}\right)\{C_1\, {}_1F_1[(1 + ia^2)/4; \tfrac{1}{2}; -i\xi^2]$$
$$+ C_2\, {}_1F_1[(3 + ia^2)/4; \tfrac{3}{2}; -i\xi^2] ,$$

[15] The magnetic field is taken to be so weak that its pressure can be neglected but not so weak that the radius of gyration of the cosmic-ray particles exceeds the scale L of the system. The range $B = 10^{-6}$–10^{-9} gauss would be a reasonable compromise in this respect.

where

$$\alpha^2 \equiv \frac{V_A}{g_0 \tau} \frac{(1+1/\Omega^2\tau^2)}{(1-1/\Omega^2\tau^2)^{1/2}}, \qquad \xi \equiv kz\left(\frac{g_0\tau}{V_A}\right)^{1/2}\left(1-\frac{1}{\Omega^2\tau^2}\right)^{1/2},$$

and $\Omega \equiv kV_A$. When $\Omega\tau > 1$, both solutions vanish at $\xi = \pm\infty$. Hence there are unstable solutions for all $\Omega\tau > 1$. When $\Omega\tau < 1$, there are no solutions satisfying the boundary conditions at $\xi = \pm\infty$. At small z,

$$\delta A = \exp\left(\frac{t}{\tau}+iky\right)\Big\{C_1\left[1+\frac{\alpha^2\xi^2}{2}+\frac{(\alpha^4/2-1)}{12}\xi^4+\ldots\right]$$
$$+C_2\xi\left[1+\frac{\alpha^2\xi^2}{6}+\frac{(\alpha^4/6-1)}{20}\xi^4+\ldots\right]\Big\}.$$

At large $|z|$, the z-dependence is made up of waves of the form $\xi^{-1/2}\exp[\pm(i/4)(2\xi^2-\alpha^2\ln\xi^2)$ when $\Omega\tau > 1$.

For the case that the cosmic-ray pressure is non-negligible, the solution is readily carried out for a cold gas $u^2 = 0$ and weak magnetic field ($B^2/8\pi \ll P$), giving

$$\frac{\partial^4\delta P}{\partial t^4}+\frac{dg}{dz}\frac{\partial^2\delta P}{\partial t^2}+g^2(z)\frac{\partial^2\delta P}{\partial y^2}=0.$$

In this case the waves at each level in the medium may have independent frequencies and wavelengths, provided only that δP remains single valued so that the equations still apply. The reason is that neither the gas nor the field transmits pressure. The cosmic-ray pressure is always uniform along each line of force in the present linear approximation. So there are no pressure fluctuations in the linear approximation for one level to disturb another. There are solutions of the form

$$\delta P = f(z)\exp\left[\frac{t}{\tau(z)}+ik(z)y\right],$$

where $f(z)$ is an arbitrary single-valued continuous function of z. The functions $k(z)$ and $\tau(z)$ are related by

$$k^2(z) = \frac{1}{g^2(z)\tau^2(z)}\left[\frac{1}{\tau^2(z)}+\frac{dg}{dz}\right].$$

Since $dg/dz > 0$, it is evident that there is always a wavenumber $k(z)$ that grows with any given rise time $\tau(z)$. The shorter the rise time, the shorter the associated wavelength. Generally speaking, $\tau(z)$ is of the order of the free-fall time over one wavelength.

APPENDIX V

STABILITY OF A CIRCULAR GEOMETRY

Consider the stability of a two-dimensional atmosphere with circular symmetry, involving a magnetic field $B(\varpi)$ whose lines of force form concentric circles about the origin. Here ϖ represents distance measured from the origin and ϕ represents azimuth measured around the origin. The gravitational field is radially inward with magnitude $g(\varpi)$. The thermal gas is isothermal with rms thermal velocity u in any given direction, so that $p = \rho u^2$. The cosmic-ray gas pressure is P. Equilibrium requires that

$$\frac{d}{d\varpi}\left(p+\frac{B^2}{8\pi}+P\right)+\frac{B^2}{4\pi\varpi} = -\rho(\varpi)g(\varpi). \qquad \text{(V.1)}$$

The simple case that the thermal gas, the magnetic-field, and the cosmic-ray gas pressures are in a fixed ratio, defined by equation (2) in the text, permits equation (V.1) to be written

$$\frac{1}{\rho}\frac{d\rho}{d\varpi}=-\frac{g(\varpi)}{u^2(1+\alpha+\beta)}-\frac{2\alpha}{(1+\alpha+\beta)\varpi}\equiv -\frac{1}{L}. \tag{V.2}$$

Introduce a transverse perturbation v_ϖ, v_ϕ with the magnetic perturbation described by the vector potential $\boldsymbol{e}_z\delta A(\varpi,\phi)$. The linearized perturbation equations may then be written

$$\frac{\partial\delta A}{\partial t}=v_\varpi B(\varpi), \tag{V.3}$$

$$\rho\frac{\partial v_\varpi}{\partial t}=-\frac{\partial}{\partial\varpi}(\delta p+\delta P)+\frac{B}{4\pi}\nabla^2\delta A+\frac{1}{4\pi\varpi}\frac{d}{d\varpi}(\varpi B)\frac{\partial\delta A}{\partial\varpi}-g\delta\rho, \tag{V.4}$$

$$\rho\frac{\partial v_\phi}{\partial t}=-\frac{1}{\varpi}\frac{\partial}{\partial\phi}(\delta p+\delta P)+\frac{1}{4\pi\varpi}\frac{d}{d\varpi}(\varpi B)\frac{1}{\varpi}\frac{\partial\delta A}{\partial\phi}, \tag{V.5}$$

$$\frac{\partial\delta\rho}{\partial t}+v_\varpi\frac{d\rho}{d\varpi}+\rho\left[\frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi v_\varpi)+\frac{1}{\varpi}\frac{\partial v_\phi}{\partial\phi}\right]=0, \tag{V.6}$$

$$\frac{\partial\delta p}{\partial t}+v_\varpi\frac{dp}{d\varpi}+\gamma u^2\rho\left[\frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi v_\varpi)+\frac{1}{\varpi}\frac{\partial v_\phi}{\partial\phi}\right]=0, \tag{V.7}$$

$$\frac{\partial\delta P}{\partial t}+v_\varpi\frac{dP}{d\varpi}=0. \tag{V.8}$$

The general solution of these equations is difficult because of the radial dependence. We know that the thermal gas is stable by itself if $\gamma > 1$. The question is whether the magnetic field and the cosmic-ray gas tend to produce instability in this configuration, as they did in the horizontal field of Appendix III. So it is sufficient to consider the thermal gas to be cold, $u^2 = 0$, thereby exposing the basic instabilities, if any, of the magnetic field and cosmic-ray gas.

Consider first the stability of the magnetic field. Put $\beta = 0$ and $\alpha u^2 = \frac{1}{2}V_A{}^2 \neq 0$, where V_A is the Alfvén speed $B/(4\pi\rho)^{1/2}$. The quantities p, P, δp, and δP vanish in the equations. Divide expression (V.6) by ρ and differentiate with respect to time. Use expression (V.3) to eliminate v_ϖ and expression (V.5) to eliminate v_ϕ from the result, obtaining

$$\frac{\partial^2}{\partial t^2}\frac{\delta\rho}{\rho}=-\frac{1}{B}\left[\left(\frac{1}{\varpi}+\frac{1}{\rho}\frac{d\rho}{d\varpi}\right)\left(\frac{\partial^2}{\partial t^2}+\frac{V_A{}^2}{\varpi^2}\frac{\partial^2}{\partial\phi^2}\right)+\frac{\partial^3}{\partial\varpi\partial t^2}\right]\delta A.$$

Then divide equation (V.4) by ρ and differentiate twice with respect to time. After eliminating $\delta\rho/\rho$, one can write the result as

$$\left[\left(\frac{\partial^2}{\partial t^2}-V_A{}^2\nabla^2\right)\frac{\partial^2}{\partial t^2}+\frac{g^2}{V_A{}^2}\left(\frac{\partial^2}{\partial t^2}+\frac{V_A{}^2}{\varpi^2}\frac{\partial^2}{\partial\phi^2}\right)\right]\delta A=0 \tag{V.9}$$

with the aid of the equilibrium relation (V.2), which reduces to

$$\frac{1}{2\rho}\frac{d\rho}{d\varpi}+\frac{1}{\varpi}=-\frac{g}{V_A{}^2}$$

in the present special circumstances.[16] A solution of the form

$$\delta A = R(\varpi) \exp i(\omega t + n\phi) \tag{V.10}$$

leads to

$$\frac{d^2R}{d\varpi^2}+\frac{1}{\varpi}\frac{dR}{d\varpi}+\left[\frac{\omega^2}{V_A{}^2}-\frac{g^2}{V_A{}^4}-\left(1+\frac{g^2}{V_A{}^2\omega^2}\right)\frac{n^2}{\varpi^2}\right]R=0\,. \tag{V.11}$$

The differential equation for the radial dependence is easily solved for the cases that g is independent of ϖ, in which the system is bound in a conical potential well, or g increases proportional to ϖ, so that the system is bound in a parabolic potential well. To take the last case first, put

$$g(\varpi) = g(a)\frac{\varpi}{a}, \qquad \omega = -\frac{i}{\tau}, \tag{V.12}$$

where $g(a)$ is the gravitational acceleration of $\varpi = a$. Then at small ϖ, such that $\varpi \ll V_A\tau$, the term $g^2/V_A{}^4$ may be neglected compared to $\omega^2/V_A{}^2$ and the solution is

$$R(\varpi) \cong J_n(q\varpi)\,, \tag{V.13}$$

where

$$q=\frac{1}{V_A\tau}\left[\left(2n\frac{\frac{1}{2}g(a)\tau^2}{a}\right)^2-1\right]^{1/2}. \tag{V.14}$$

In the limit of large τ,

$$q\varpi \sim n\frac{g(a)\tau}{V_A}\frac{\varpi}{a}, \tag{V.15}$$

which may be large compared to 1 while $\varpi \ll V_A\tau$. It is evident that the unstable solution is well behaved, satisfying $\delta A = 0$ at some inner radius $\varpi = b$, say, and oscillating with declining amplitude with increasing ϖ. At large ϖ the solution declines asymptotically as $\varpi^{-1/2} \exp[-g(a)\varpi^2/2V_A{}^2a]$. Within the limitations of these boundary conditions, the system is unstable, just as in the flat atmosphere considered in Appendix III.

If g is independent of ϖ,[17] the solution of the radial equation is

$$R(\varpi) = Z_{ip}\left(i\frac{\varpi}{l}\right), \tag{V.16}$$

where Z represents a Bessel function and

$$\frac{1}{l}=\left(\frac{g^2}{V_A{}^4}+\frac{1}{V_A{}^2\tau^2}\right)^{1/2}, \qquad p = \pm n\left(\frac{g^2\tau^2}{V_A{}^2}-1\right)^{1/2}. \tag{V.17}$$

In the limit of large τ, $l \sim V_A{}^2/g$ and $p \sim ng\tau/V_A{}^2$. The solution

$$R(\varpi) = i[J_{ip}(i\varpi/l) - \exp(-\pi p)J_{-ip}(i\varpi/l)] \tag{V.18}$$

is real and well behaved at all finite ϖ, going to zero as $\varpi \to \infty$. This is readily seen from the leading terms of the expansion at small ϖ/l.

$$R(\varpi) \cong \frac{\exp(-\pi p/2)}{P}\left[\frac{\exp(ip\ln\varpi/2l)}{\Gamma(ip)}+\frac{\exp(-ip\ln\varpi/2l)}{\Gamma(-ip)}\right], \tag{V.19}$$

[16] Integration of this equation with $g(\varpi) = g_0$ and $g(\varpi) = g_0\varpi/a$ leads to $\rho(\varpi) = \rho(a)(a/\varpi)^2 \exp(-2g_0\varpi/V_A{}^2)$ and $\rho(a)(a/\varpi)^2 \exp(-g_0\varpi^2/V_A{}^2a)$, respectively. The singularity ϖ^{-2} at the origin is artificial, introduced by the failure of the simple form $B^2 \propto p$ to give $B \to 0$ at $\varpi = 0$. The singularity is readily excluded by inclosing it with a circular boundary, $\varpi = b$.

[17] There is a discontinuity in the gravitational field at the origin in this case.

which oscillates rapidly, and from the asymptotic expansion

$$R(\varpi)\sim\exp\left(\frac{\pi p}{2}\right)[1-\exp(-2\pi p)]\left(\frac{l}{2\pi\varpi}\right)^{1/2}\exp\left(-\frac{\varpi}{l}\right) \tag{V.20}$$

at large ϖ/l. Thus again the system is unstable for large $\tau(g\tau \gg V_A)$.

Consider the other example now in which the magnetic-field pressure can be neglected but the cosmic-ray gas pressure is finite. Then $\delta A = 0$, $\delta p = 0$. Differentiate expression (V.6) with respect to time and use formulae (V.4) and (V.5) to eliminate v_ϖ and v_ϕ. Differentiate expression (V.8) with respect to time and use expression (V.4) to eliminate v_ϖ, solving the result for $\delta\rho$, which may then be used to eliminate $\delta\rho$ from the previous equation. The resulting equation is

$$\frac{\partial^4\delta P}{\delta t^4}+\left(\frac{dg}{d\varpi}-\frac{g}{\varpi}\right)\frac{\partial^2\delta P}{\partial t^2}+\frac{g^2}{\varpi^2}\frac{\partial^2\delta P}{\partial\phi^2}=0\,. \tag{V.21}$$

This equation has solutions of the form

$$\delta P=h(\varpi)\exp\left[\frac{t}{\tau(\varpi)}+in(\varpi)\phi\right],$$

where $h(\varpi)$ is an arbitrary single-valued function of ϖ. However, with a circular geometry $n(\varpi)$ is not an arbitrary function of its argument because δP must be a single-valued function of ϕ, requiring that $n(\varpi)$ equal an integer. Since the displacement of the medium must also be a single-valued continuous function of ϖ, it follows that $n(\varpi)$ cannot change discontinuously, so n must have the same integral value for all ϖ. Consequently, $\tau(\varpi)$ is determined by n and $g(\varpi)$ from

$$\frac{1}{\tau^4(\varpi)}+\frac{1}{\tau^2(\varpi)}\left(\frac{dg}{d\varpi}-\frac{g}{\varpi}\right)-\frac{n^2g^2}{\varpi^2}=0\,. \tag{V.22}$$

The growth time $\tau(\varpi)$ is independent of ϖ only for the special form of $g(\varpi)$ given by

$$g(\varpi)=\frac{\varpi}{n\tau^2}\left(\frac{c^{2n}-\varpi^{2n}}{c^{2n}+\varpi^{2n}}\right), \tag{V.23}$$

where c is a constant. For $c\rightarrow+\infty$, the gravitational field becomes a parabolic potential well. If we write $g(\varpi) = g_0\varpi/a$, we get $\tau^2 = a/ng_0$ as the characteristic growth time in this case. This is again of the order of the free-fall time over one wavelength.

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