Calibration of the sign of Stokes V with XPOL

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Abstract

We investigate the calibration of the sign of Stokes V in observations with XPOL at the IRAM 30m telescope. Three methods are used and described in full detail: (1) the sign is derived from first principles and shown to agree with the usage in VLBI measurements, (2) unpolarized emission from planets is converted to circular of known sense with the help of a wire grid/quarterwave plate assembly, and (3) the linearly polarized flux of the Crab nebula is converted to circular of known sense with a quarterwave plate. All 3 methods agree and give *negative* Stokes V for RHC polarization. This result is opposite to IAU convention, and the next version of the XPOL analysis software will reverse the sign of Stokes V (positive V corresponds to RHC).

Introduction

Following IAU prescriptions[3] for astronomical polarization observations, we employ a right–handed coordinate system $\mathcal{K_S}$ whose origin is located in the observed target on the sky. The x, y, and z axes of $\mathcal{K_S}$ point to the celestial pole, respectively, towards increasing right ascension, and z towards the observer. We adopt a second coordinate system $\mathcal{K_N}$ fixed in the telescope's Nasmyth receiver cabin (Fig. 1) where the polarization measurement is made. The systems $\mathcal{K_N}$ and $\mathcal{K_S}$ differ only by a trivial translation, an inconsequential even number (6) of reflections, and by a rotation about the z–axis. The Stokes parameters measured in $\mathcal{K_N}$ are then related to $\mathcal{K_S}$ by a simple rotation.

The calibration of the Stokes parameters I and Q, the sum and the difference, respectively, of the powers detected by A100 and B100, is trivial. Stokes U and V are measured by XPOL as the complex cross correlation between these two receivers. We determine the sign of U through an observation of the Crab Nebula whose strong linear polarization, $p_L = 25\%$ permits an accurate measurement of its polarization angle $\chi = 155°$ [5]. The sign of Stokes V is less easily determined, since there is no strong circular polarization calibrator on the millimeter sky. Here we solve this problem for the 30m telescope by (1) analysing the transport of polarization through the telescope from basic principles, by (2) converting the flux from strong unpolarized sources to circular polarization of a known sense, and (3) by converting the strong linearly polarized flux of the Crab Nebula into circular polarization of a known sense.

Description of circularly polarized waves

In the absence of linear polarization a monochromatic electromagnetic wave is described by sinusoidal oscillation of its E–field in orthogonal directions x and y. To fix our ideas, we adopt:

$$E_x = E_0\, cos(\omega\, t) \qquad\qquad E_y = E_0\, sin(\omega\, t) \tag{1}$$

where ω is the circular frequency of the wave. We see that the y oscillation is lagging behind the x component by 90°. In a right–handed coordinate system (Fig. 2) the addition of these oscillations produces a counter–clockwise (ccw) circular motion of the E–vector whose position angle is thus always increasing.

In a right–handed expansion of this coordinate system into 3 dimensions, we have the $+z$–axis pointing out of the paper in the direction of the reader. If we add to the ccw circular motion of E a uniform speed (velocity of light c) in the $+z$–direction, we have a circularly polarized electromagnetic wave whose E–vector moves on a right–handed screw towards the observer. This wave is said to be righthand circularly (RHC) polarized [2]. Its angular momentum vector points in the $+z$–direction, its direction of motion, and the wave is therefore said to have positive helicity[6].

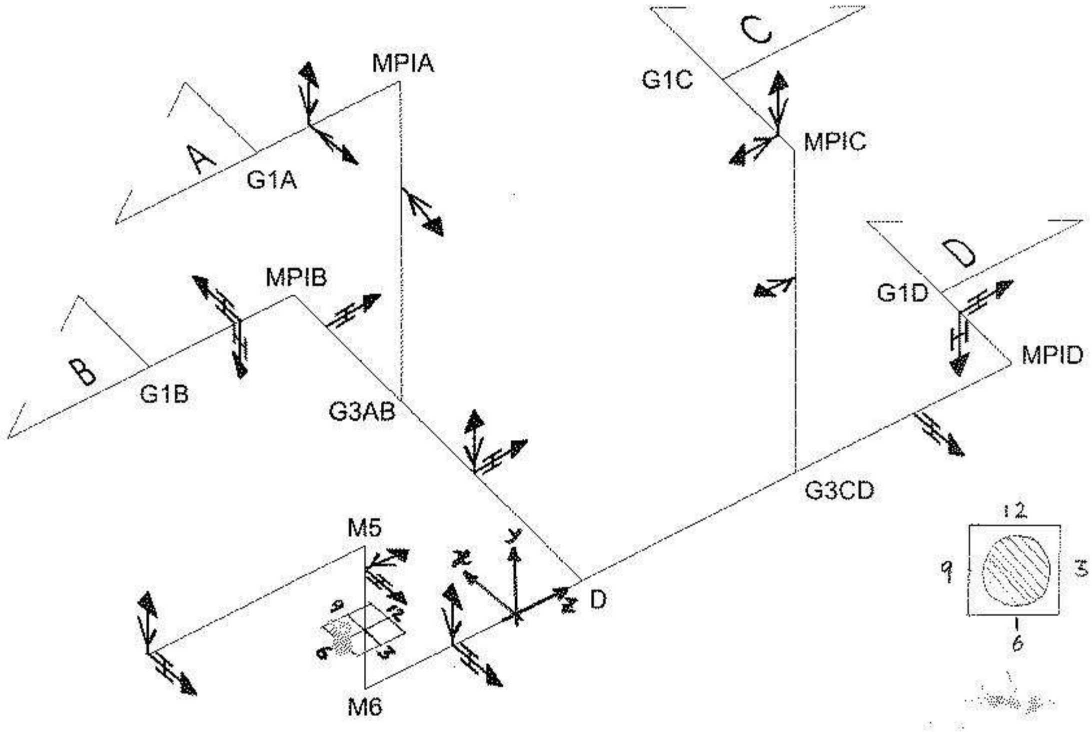

Figure 1: Nasmyth receiver cabin optics. The incoming beam (lower left) is relayed through mirrors M5 and M6 to the beam divider D. In 3mm experiments, it is set to reflect the unpolarized incoming beam through various grids and Martin–Puplett Interferometers to the two receivers, A100 (vertically polarized) and B100 (horizontally polarized). The pairs of bold arrows labelled H and V indicate the transport of horizontal and vertical linear polarization. The VLBI plate, QWP, is located horizontally between mirrors M5 and M6. The grooves of the QWP are oriented along the 4:30/10:30 o'clock line (inset). Starting from the plate, the telescope beam goes through 5 reflections before reaching the sky. (The drawing was prepared by B. Lazareff, IRAM receiver group)

In summary, using unit vectors $\vec{e}_x$ and $\vec{e}_y$ and k being the wave number, we have:

$$RHC: \quad E = E_0\,(\,cos\,(\omega\,t - kz)\,\vec{e}_x \;+\; sin\,(\omega\,t - kz)\,\vec{e}_y\,) \tag{2}$$

$$LHC: \quad E = E_0\,(\,cos\,(\omega\,t - kz)\,\vec{e}_x \;-\; sin\,(\omega\,t - kz)\,\vec{e}_y\,) \tag{3}$$

The LHC wave can be thought to arise from the circular motion depicted in Fig. 2 by inverting the oscillation in the y–direction.

The x–component of the RHC wave is shifted relative to the y–component by a quarter period to earlier times t. The x–component is said to be advanced with respect to the y–component.

These conventions and conclusions agree with the IAU definitions on Polarimetry[3, 1] which furthermore defines Stokes V as

$$V = RHC - LHC$$

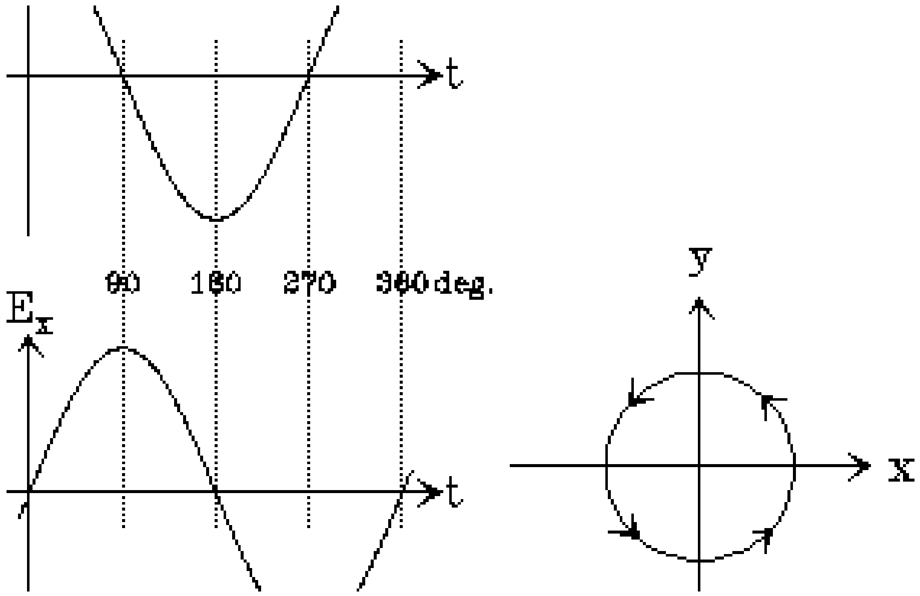

Figure 2: Sinusoidal oscillations of the electric field E in the x and y directions. The y component, $sin(\omega\,t)$, is retarded by a quarter period with respect to the x component, $cos(\omega\,t)$. In superposition, the two oscillation form a counterclockwise circular motion.

In summary, if looking into an oncoming wave, the E–vector is rotating counterclockwise (in a righthanded screw), the wave is said to be RHC polarized. Its E–vector varies like $ei\omega\,t$, and in a right handed system, the wave's y component is retarded with respect to the x component.

Quarter wave plates

Quarter wave plates (QWP) figure prominently in polarization work, and they are also employed here. In particular, we make use of a QWP manufactured by IRAM for VLBI observations at 86 GHz. This plate is machined out of high refractive index plastic material. Both sides of the plate have parallel grooves which give the plate an unisotropic transmission. The refractive index is higher for waves linearly polarized parallel to the grooves than perpendicular to them. A wave therefore travels slower (faster) through the plate if polarized parallel (perpendicular) to the grooves. The optical design and the manufacture of the plate is described by J. Lamb[4].

Fig. 3 shows the passage of a RHC wave through a QWP whose grooves are oriented parallel to the x–axis. The receiver Rx looks into the oncoming wave. The same right–handed coordinate system is used as in Fig. 2 and the wave has the same x and y components where x is advanced with respect to y. Since the QWP presents its slow axis to the incident x–component, the plate delays the advanced component by a quarter period. The transmitted x and y components are therefore in phase and thus combine to form a linearly polarized wave whose polarization angle is at 45°. As usual in right handed systems, the angle is counted clockwise from the x–axis.

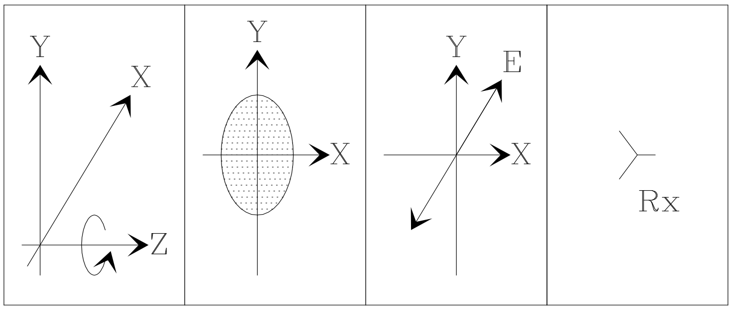

Figure 3: A RHC polarized wave incident on a quarter wave plate. If the plate is oriented with its slow axis (grooves) parallel to the x–axis, RHC is coverted to linear polarization with the polarization angle at 45° as viewed from the receiver Rx.

The action of a circularly polarized wave incident on a quarter wave plate, as viwed from the receiver, is therefore summarized as follows (also depicted in Fig. 4), looking as always into the oncoming wave:

incoming		outgoing linear at angle
RHC	$\Longrightarrow$	45° ccw from grooves
LHC	$\Longrightarrow$	45° cw from grooves

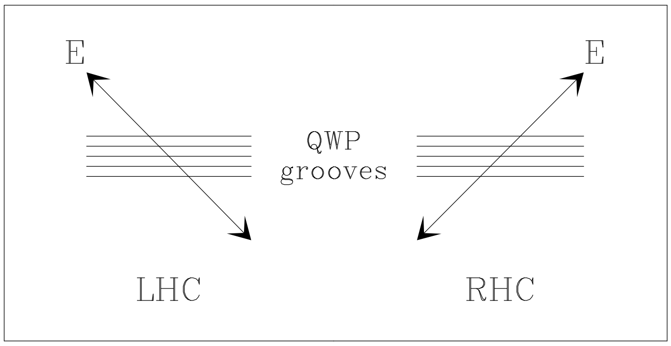

Figure 4: Orientation of the E–vector after passage of a circularly polarized wave through a quarter wave plate QWP. View is into the oncoming wave. The slow axis is parallel to the grooves.

VLBI observations

The IRAM 30m telescope participates in global 3mm VLBI observations since many years. These observations routinely use LHC receivers. In VLBI sessions, the linear receivers of the 30m telescope are converted to circular by inserting a quarter wave plate into the beam common to the two 3mm receivers A100 and B100. Correlations with other VLBI telescopes has established the sense of polarization of the 30m receivers A100 (LHC) and B100 (RHC). Here we derive this identification from the "first principles" outlined above. The success of this identification gives confidence that we understand the transport of polarization through the telescope when applied to XPOL below.

The two 3mm receivers have orthogonal linear polarizations: vertical (V) for A100 and horizontal (H) for B100. These orientations refer to the receiver cabin as depicted in Fig. 1 which shows the transport of H and V through the various optical components of the Nasmyth cabin. The VLBI plate is inserted into the beam in a horizontal plane[1] between mirrors M5 and M6. Looking down from M5 onto the plate, its grooves are oriented in the 4:30/10:30 o'clock direction (see inset of Fig. 1), i.e. 45° counter clockwise from the local V. When taking our usual viewing direction into the oncoming wave (from the receiver toward the sky), the grooves appear at 45° clockwise from V. Consultation of Fig. 4 shows that the plate

[1]This plane is actually slightly tilted ($\sim 5°$) in order to reduce standing waves.

converts V–polarization into RHC. A righthand circular beam incident on the plate is therefore converted into vertical linear polarization and detected by receiver A100. Between the plate and the sky there are however 5 more reflections (Nasmyth mirrors 3,4, and 5; the subreflector and the main reflector). As a consequence, a RHC wave incident on the plate originates as a LHC wave on the sky.

We have thus derived the polarization/receiver correspondence observed in 3mm VLBI experiments, i.e. with the 86 GHz VLBI quarterwave plate inserted according to Fig. 1

A100	V	LHC
B100	H	RHC

Observing fake circularly polarized sources

Unfortunately, there are no strongly circularly polarized and well calibrated sources on the mm sky. We remedied this situation by converting the unpolarized continuum from Mars and Venus into RHC and LHC. A wire grid/quarterwave plate assembly was introduced into the telescope beam. at the same position as the VLBI plate (Fig. 1). The QWP was used in the same orientation calibrated for VLBI observations.

Two observations were made of each planet where the wire grids were oriented with their wires at $\pm 45°$ with respect to the grooves of the waveplate. When looking down from M5 onto the grid/QWP assembly, we note that, e.g., the 6 – 12 o'clock line appers at $\Delta\chi = +45°$ from the grooves. The angle changes to $\Delta\chi = -45°$ when viewed from the receiver. Consulting the QWP primer (Fig. 4) we conclude that a wave polarized linearly in the 6–12 direction (grid oriented with its wires along 3 – 9 o'clock) is converted into RHC. Since there is an odd number of reflections between the position of the grid/QWP assembly and the sky, the assembly with its wires in the 3 – 9 (6 – 12) o'clock orientation mimickes LHC (RHC) polarized radiation from the sky.

Our observations of Venus and Mars give compatible and unambiguous results (Tab. 1 and Fig. 5). Our grid/QWP assembly gives indeed within measurement accuracy 100% circular polarization, despite the strong standing waves which are due to the strong source continuum and the severe obstruction presented by the grid/QWP assembly. Particularly the massive metal mounting plate of the grid covered a large part of the beam. The main result is however clear: RHC appears to correspond to *negative V*.

The experiment on Venus was in fact made in a non–standard mode where the grid which splits the incoming beam between receivers A100 and B100 (labelled G3AB in Fig. 1) was rotated by 90°. In its rotated position where drastically lower polarization sidelobes are observed[5], the polarization of the receivers is exchanged and the sign of quadrature cross correlation is reversed. This sign reversal is actually observed, and it is taken into account by the XPOL analysis software.

Observation of the Crab Nebula

Instead of converting a fully unpolarized source into circular polarization of a known sense, we here observed the stronly linearly polarized Crab nebula ($p_L = 25\%$ at 86 GHz[5]). Due to its sidereal motion its Stokes parameters Q and U oscillate strongly in the Nasmyth cabin. Stokes Q reaches positive and negative maxima at hour angles -1.00 and $+1.55$. At these hour angles all linearly polarized power is in the Nasmyth vertical (HA =+1.00) or horizontal (HA = -1.55) plane. With the QWP inserted in its usual position and orientation, the observation at HA = +1.00 (-1.55) sees the Crab nebula in LHC (RHC) light. The sign of the observed Stokes V agrees with this expectation (Tab. 1).

Conclusion

We have used three, to a large degree independent methods to derive the sign of Stokes V in polarimetric observations with XPOL at the IRAM 30m telescope. All methods agree in giving *negative* Stokes V for incoming RHC polarized radiation with the current version 3.2 of the XPOL analysis software. As this sign is opposite to the IAU convention[3], the sign will be reversed (RHC giving positive Stokes V) in Vesion 3.3 of the XPOL analysis software.

Although XPOL works at any wavelength accessible to the 30m receivers, the calibration of the Stokes V sign described here was only made at 86 GHz. Polarimetry at frequencies outside the 3mm

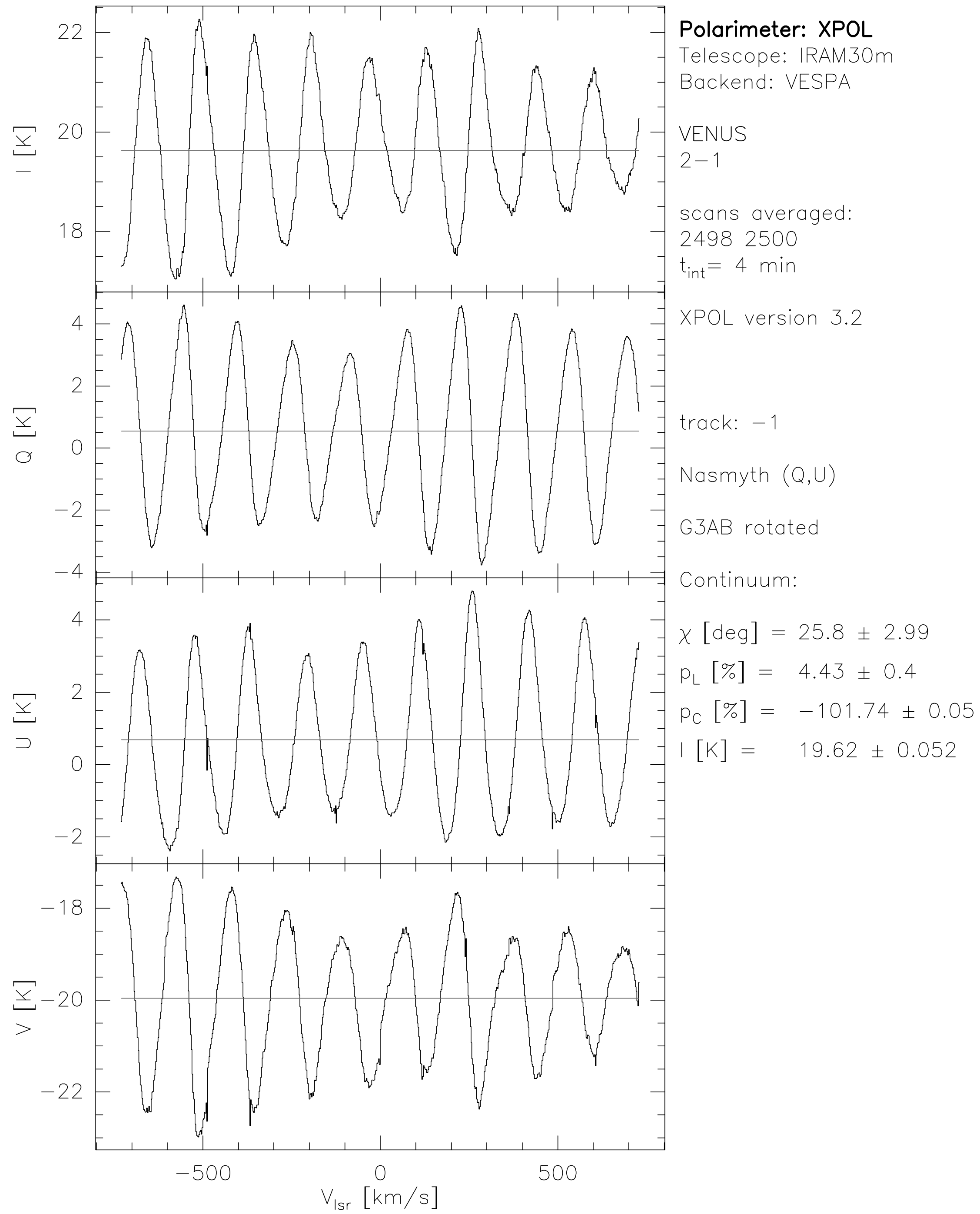

Figure 5: XPOL observation of Venus. The grid/QWP assembly was inserted in its RHC position. Standing waves of ca. 10% amplitude are caused by the strong source continuum and the considerable obstruction of the beam by the grid/QWP assembly. Small linear polarization is due to imperfections of grid and QWP, and due to the non/orthogonal mounting of the assembly in the beam.

Table 1: Observations made of Mars, Venus, and the Crab Nebula. For the planets the setup indicates the angle $\Delta\chi$ between grooves and wires as viewd from the receiver, for the Crab we indicate the hour angle of the observation (see text).

source	setup	XPOL	expectation	remark
Mars	45° cw	V = +100%	LHC	G3 standard
Mars	45° ccw	V = −100%	RHC	G3 standard
Venus	45° cw	V = +100%	LHC	G3 rotated
Venus	45° ccw	V = −100%	RHC	G3 rotated
Crab	HA = +1.00	V = +25%	LHC	G3 rotated
Crab	HA = −1.55	V = −25%	RHC	G3 standard

window are carried out with other receiver, possibly with different polarizations in the reference system $\mathcal{K}_{\mathcal{N}}$. XPOL analysis software will be upgraded to handle these situations.

References

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