Estimation of the NIKA v2 optics grid distortion from empirical fits on a Zemax simulation

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Purpose: model the simulated image distortion in order to have a reference to compare with the real data and possibly use it to improve the data reduction.

The Zemax distortion grid:
"Zemax computes the coordinates of a grid of chief rays. In a system without distortion, the chief ray coordinates on the image surface follow a linear relationship with the field coordinate. To compute this linear relationship, ZEMAX traces rays over a very small region centered upon the reference field position (which is the center of the field of view). The grid distortion plot shows the linear grid, and then marks the actual chief ray intercept for a ray with the same linear field coordinates with for each point on the grid."

Zemax does not compute the equations that allows to transform the undistorted grid to the distorted grid and vice versa. So I grabbed the Zemax grid distortion and found by empirical fitting a set of equations allowing to perform this transformation.
From the plots shape of Zemax (see figure below) it is obvious that the main distortion is a cushion (which create a barrel-distorted image when the regular grid array is projected back to the sky). But this is not the only distortion. Automatic polynomial fits on the residual components don't give satisfactory results in terms of equation sizes, so to find a good fit using the minimum number of extra terms I empirically deduced the main extra components from the observation of the curves of the residuals obtained after subtraction of the main barrel correction.

Equations allowing to transform the real grid values (Xr, Yr) into the calculated grid (Xc, Yc) fitting the predicted linear (Xp, Yp):

$Xc = Xr (1 + a*(Xr^2 + Yr^2)) + b*Yr^2 + c*Xr + d*Xr^3*Yr^4$
$Yc = Yr (1 + a*(Xr^2 + Yr^2)) + b*Yr*\exp((Xr*c-d)^2) + e*Yr^3*Xr^2$

Values of the parameters:

	a (barrel)	b	c	d	e
X	$-5.58\ 10^{-4}$	$-7.72\ 10^{-4}$	$-6.63\ 10^{-3}$	$0.97\ 10^{-8}$	
Y	$-5.62\ 10^{-4}$	$-1.03\ 10^{-5}$	$2.39\ 10^{-2}$	2.53	$1.72\ 10^{-6}$

The barrel distortion is not square but slightly rectangular; this is due to the biconic mirror, and the other terms are due to the off-axis optics. Note that a careful optics design allows to minimize greatly these distortions.

The table below display the least-squares of the residuals ($\Sigma((Xc - Xp)^2)/\Sigma(Xp^2)$), using the barrel only, or the barrel plus part of the extra terms or all the extra terms:

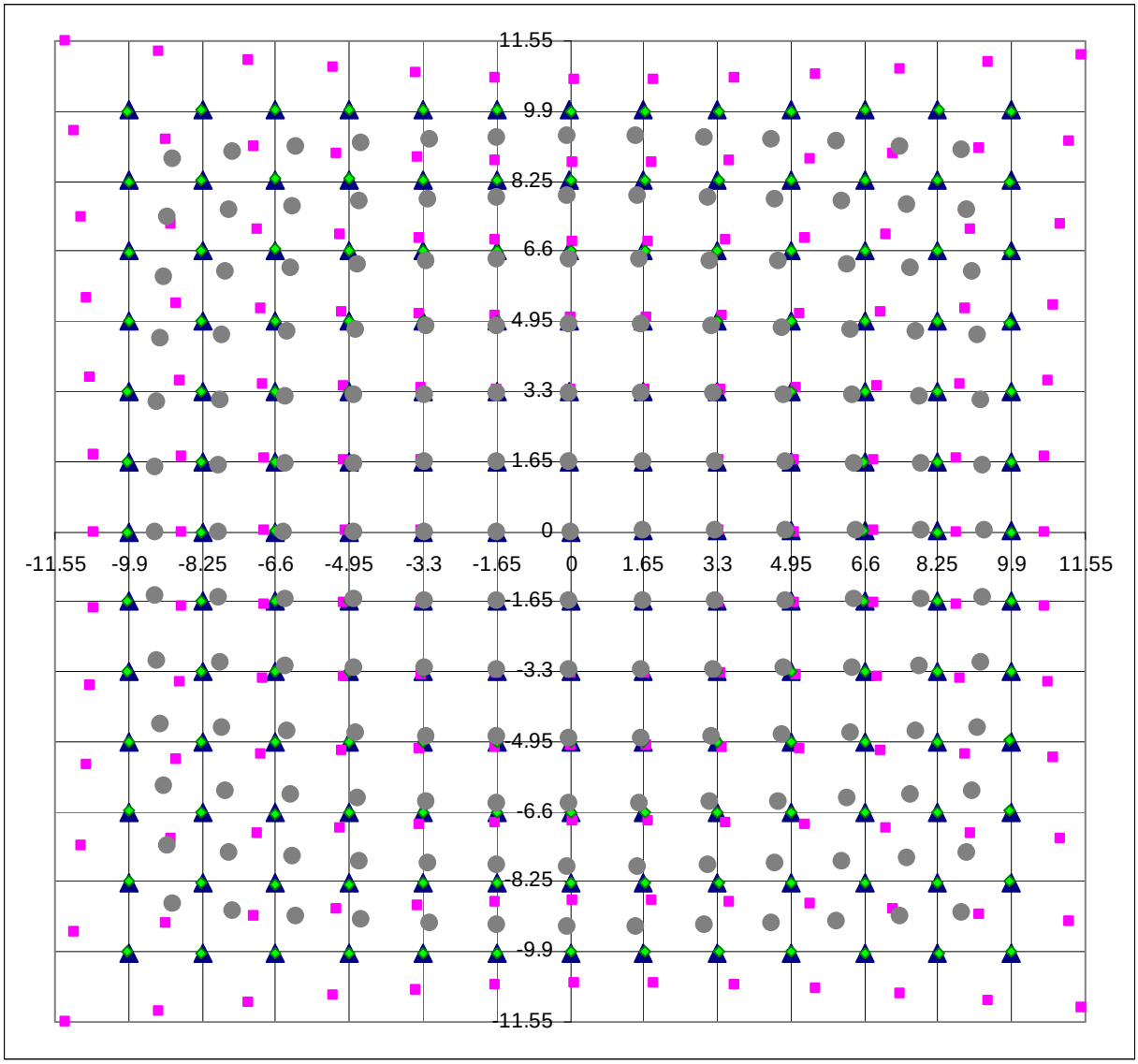
X	a only: 1.08 %	a, b: 0.74 %	a, d, c, d: 0.25 %
Y	a only: 1.07 %	n/a	a, b, c, d, e: 0.22 %

Note: for Xc if only the 2nd or the 3rd extra term are present the least-square is worse, for Yc if only the 1st and 2nd term is present the least square is worse.

The Zemax predicted linear and real grids with the corrected grids obtained with my fits are displayed on the plot below:

- Blue triangle = (Xp, Yp) = linear grid calculated by Zemax from a small region at the center of the image.
- Pink squares = (Xr, Yr) = positions on the image plane of the “real” chief rays passing through the optics according to the Zemax model of the NIKA optics (+ 30m telescope).
- Green diamonds = (Xc, Yc) = calculated grid after transformation of the Zemax “real” grid with the equations defined on the previous page.
- Grey dots = (Xd, Yd) = calculated grid after transformation of the Zemax linear grid with the equations defined on the previous page; (Xd, Yd) should be close to the grid distortion observed on the data.

The units on the plots are millimeters on the image plane (NIKA array). In terms of fields angles in the sky, zemax gives the correspondance: 9.9 mm on the image = 1.37*10-2 degrees on the sky => the FOV of the diagonal is 2.33 arc minutes.



Equations allowing to do the inverse transformation (calculating (Xc,Yc) from (Xp,Yp) such that (Xc,Yc) fit (Xr,Yr)):

Xc = Xp + a*(Xp^3 + Xp*Yp^2) + b*Yp^2
Yc = Yp + a*(Yp^3 + Yp*Xp^2) + b*Xp*Yp

Values of the parameters:

	a (cushion)	b			
X	$7.65\ 10^{-4}$	$9.85\ 10^{-4}$			
Y	$7.61\ 10^{-4}$	$-15.1\ 10^{-5}$			

Least-squares of the residuals ($\Sigma((Xc - Xr)^2)/\Sigma(Xr^2)$), using the cushion only, or the barrel plus part of the extra terms or all the extra terms:

X	a only: 0.78 %	a, b: 0.18 %	
Y	a only: 0.86 %	a, b: 0.05 %	

The Zemax predicted linear and real grids with the corrected grid obtained with my fit are displayed on the plot below:

- Blue triangle and pink squares = as previous plot
- Green diamonds = (Xc, Yc) = calculated grid after transformation of the Zemax predicted grid with the equations defined above.