GISMO sensitivity at Pico Veleta
Feedback on Dominic Benford "Atmospheric Emission at Pico Veleta"
Samuel Leclercq, June/August 2009

Introduction

The GISMO team and myself at IRAM calculated independently the theoretical optimal sensitivities (background sky and optics dominating the instrument noise) of GISMO to serve as a reference for comparison with the actual sensitivities on astronomical observations. The numbers being different, Dominic Benford and Johannes Staguhn sent a report of their detailed calculations to IRAM for examination. But some values, equations and other elements in Dominic's document remains unclear. So I wrote the present document in which I compare both calculations including all material necessary for reader to check every step, understand the problematic and possibly contribute to the discussion. Hopefully we will reach consensus for these estimations. **The questions left unanswered after reading Dominic's document are highlighted with bold teal font**.

1) Atmosphere

Dominic used the Harvard-Smithsonian AT model to simulate the emissivity of the atmosphere at Pico Veleta. I used the ATM model from the GILDAS package developed at IRAM. The curves from both models don't seem to match very well. So to get stand-alone calculations easy to check and exchange I created a simplified model with the following constraints:

(1) The domain of validity must cover IRAM's bands: $50 \le \nu \, [\text{GHz}] \le 400$
(2) The simple model must fit ATM with errors $\Delta\tau(\nu)/\tau < 4\%$ in the atmospheric windows
(3) The errors can be high for individual lines in the atmospheric opacity "walls", but always $\Delta\tau(\nu)/\tau < 60\%$ and the average for each of the 5 "walls" must be $|\Delta\tau(\nu)/\tau| < 4\%$.
(4) For simplicity the model must contain a minimum number of parameters, keeping only the atmospheric features with influence on large bandwidth (>10GHz) detectors
(5) The telescope elevation and weather conditions must be tunable, but the atmosphere temperature may be set at an invariant value for simplicity of the model.

Table 1. Simple atmospheric model[1] computed for the altitude of Pico Veleta and an outdoor temperature $T_a = 275$ K. The variables are ν the frequency, and w the millimeters of water vapor in the atmosphere. The lines dependency on water vapor is: $p=0$ for O_2 and $p=1$ for H_2O.

Pseudo-continuum: $\quad \tau_c(\nu, w) = \left(a_c \cdot w + b_c\right) \cdot \left(\dfrac{\nu}{\nu_c}\right)^2$

ν_c [GHz]	a_c	b_c
250	0.071	0.005

"Kinetic" lines: $\quad \tau_l(\nu, w) = \sum_l \dfrac{w^{p_l} \cdot \tau_{o_l}}{\left(1 + \left(\dfrac{\nu_{o_l}^2 - \nu^2}{\nu_{s_l} \cdot \nu}\right)^2\right)}$

ν_0 [GHz]	ν_s [GHz]	τ_0	p
58.2	2.5	3.2	0
60.2	2	11.5	0
118.7	1	9.4	0
183.3	2.96	2.2	1
125.1	3.47	2	1
368.5	0.56	1	0
380.2	3.49	19	1

Gaussians: $\quad \tau_g(\nu, w) = \sum_g w^{p_g} \cdot \tau_{g_g} \cdot \exp\left(-\left(\dfrac{\nu_{o_g} - \nu}{\nu_{s_g}}\right)^2\right)$

ν_0 [GHz]	ν_s [GHz]	τ_0	p
58.1	2.5	17.6	0
62	2.1	20.2	0
65.3	3.1	0.2	0
440	80	0.13	1

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[1] The pseudo-continuum and "kinetic" lines (Zhevakin & Naumov) are based on equations from Cernicharo et al, IEEE ant. prop. Dec 2001. The "gaussian bunches of lines" are empirical fits allowing to reach the specifications of the model using a minimum number of true molecular lines

The global opacity, function of the frequency, water vapor and the telescope elevation θ is:

$$\tau(\nu, w, \theta) = \left(\tau_c(\nu, w) + \tau_l(\nu, w) + \tau_g(\nu, w)\right) \cdot \frac{1}{\sin(\theta)}$$

The airmass approximation ($1/\sin(\theta)$) is always valid: 0.2% error at maximum (for $\theta = 20°$).
The emissivity (ε) and transmission efficiency (t) of the atmosphere are:

$$\varepsilon = 1 - \exp(-\tau) \qquad\qquad t = \exp(-\tau)$$

As figure 1 shows, to get a "typical" curve similar to Dominic ($\varepsilon = 20\%$ @ 150 GHz) with the simple model, some possible combinations of water vapor and elevation are $[w(\text{mm}); \theta(\text{degrees})] = [3;25]$ or $[5;45]$ or $[7;75]$. **These conditions are a bit pessimistic (too low on the sky or too wet), aren't they? Or is there a discrepancy between AT and ATM models? Part of the answer requires defining typical observing conditions for GISMO.**
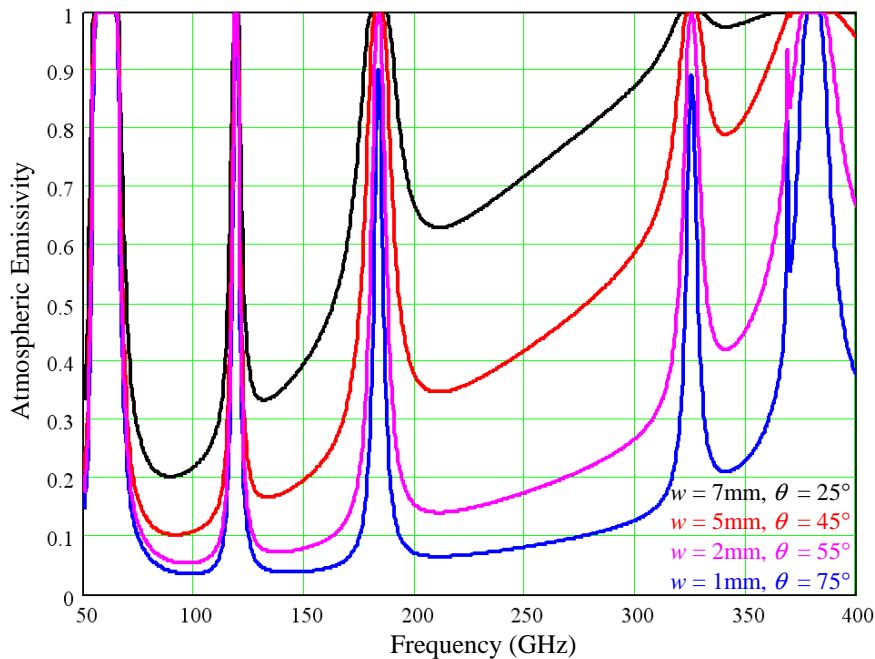

Figure 1. Atmosphere emissivity deduced from my simple opacity model. Black line = bad observing conditions = 7mm of water vapor and 25 degrees elevation, red line = tuned to get a curve similar to Dominic's typical observing conditions = 5mm of water vapor and 45 degrees elevation, magenta line = my typical observing conditions = 2mm of water vapor and 55 degrees elevation, blue line = good observing conditions = 1mm of water vapor and 75 degrees elevation. At GISMO central frequency (150 GHz) the emissivities for the bad, Dominic's, mine, and good observing conditions are respectively: 40%, 20%, 8%, 4%.

From these curves and the tau-meter data from October 2008 (see the document replying J.Staguhn report) it seems that Dominic's "typical weather" is rather typical of the run #2 bad weather rather than a typical mean weather for observation with bolometers at Pico Veleta.

2) Telescope and instrument efficiencies and other optical parameters

Lists of the telescope parameters needed for the calculation of the collected power:
- Telescope diameter $D = 30$m
- Aperture efficiency at long wavelength $\eta_{a0} = 69$ %
- Primary mirror surface deformation heights $\sigma_h = 0.055$ mm
- Correlation lengths of surface deformations $d_e = (2.5; 1.7; 0.3)$ m
- Parabolic mirror steepness factor $R = 0.9$
- Ruze parameter $R\sigma = 0.06$
- Ohmic losses for a reflection on a mirror $e_\Omega = 1$ %
- Number of mirrors (in the cabin) $n_m = 6$

- Other optical losses $\eta_o = 97\ \%$
- Typical temperature of the mirrors $T_t = 280$ K

Lists of the instrument parameters needed for the calculation of the collected power:
- GISMO central frequency (and wavelength) $v_G = 150$ GHz $(\lambda_G = 2$ mm)
- GISMO band-pass filter width $\Delta v/v = 15\ \% \Rightarrow \Delta v_G = 22$ GHz
- Pixel size relative to the diffraction pattern[2] $u_G = 0.9$ Fλ
- Pixel quantum efficiency $\eta_p = 90\ \%$
- Transmission of a filter (blocker, edge or band-pass) $t_f = 95\ \%$
- Number of filters (r = room temperature, N = liquid nitrogen temperature, He = liquid helium temperature, c = cold stage temperature) $n_r = 1, n_N = 3, n_{He} = 2, n_c = 1$
- Transmission of the 4K Si lens **t_l =90%? (see η_{sys} below)**
- Transmission of the optional neutral density filter $t_{nd} = 40\ \%$

<u>Attenuation factors calculated from the telescope and instrument parameters:</u>

- Effective aperture (Dominic versus Samuel): $A_D = 500$ m^2 $A_S = 600$ m^2
The telescope main dish area is $A_t = 707$ m^2. The vignetting plots from a Zemax simulation of GISMO in the 30m receiver cabin show that ratio of unvignetted rays is about 85 %. Dominic's effective gives a surface ratio $A_D/A_t = 71\%$; this is neither the vignetting ratio nor the aperture efficiency ($\varepsilon_0 = 69\%$ and $\varepsilon_a(150\text{GHz}) = 57\%$), **so what does the 500m^2 "effective aperture" represent exactly?**

- Throughput (Dominic versus Samuel): $A\Omega_D = 3.93$ mm^{2}sr $A\Omega_S = 2.5$ mm^{2}sr
Two equivalent calculations giving my value: (1) telescope area * angular size of the pixel in the sky: $A \cdot \Theta_b^2 = (\pi D^2/4) \cdot (u_G \cdot \lambda/D)^2$, (2) pixel area * solid angle at which it sees the pupil: $S_p \cdot \Omega_p = (u\lambda F)^2 \cdot (\pi/4F^2)$, **so why Dominic's throughput is 35 % bigger than mine?**

- System optical efficiency: $\eta_{sysD} = 35\ \%$ $\eta_{sysS} = 52\ \%$
My calculation: $\eta_{sysS} = t_t \cdot \eta_o \cdot t_f^{\Sigma n_f} \cdot t_l \cdot \eta_p$, where $t_t = (1 - e_\Omega)^{nm}$ is the telescope transmission. To find $\eta_{sysD} = 35\ \%$ using my calculation I had to make the hypothesis that **$t_{lD} = 60\%$, but this seems low isn't it? Johannes once the loss was small, so I choose $t_{lS} = 90\%$; what is the actual value of t_l? If >60%, what are the other factors responsible for such a small η_{sysD}?**

- Forward and main beam efficiencies: $\xi_D = 88\ \%$, $\eta_{MBD} = 68\ \%$ $\xi_S = 90\ \%$, $\eta_{MBS} = 60\%$
My calculations are based on the antenna tolerance theory[3] (ATT) using surface deformations ($\eta_{a0}, R, d_e, \sigma_h$). To avoid overloading the document with the details of ATT, I only show the links between the degraded beam pattern I_t, the main and error beams I_m and I_e, the relative power L_n and the efficiencies η, as well as the Ruze laws for the aperture and beam efficiencies η_a and η_{mb}, and a fit to the IRAM web site values for the forward efficiency η_F:

$$I_t = I_{mb} + \sum I_e \qquad L_n = \int_r I_t(\rho)d\rho \Big/ \int_\infty I_t(\rho)d\rho \qquad \eta_x = L_n(x)$$

$$\eta_a(\lambda) = \eta_{a0} \cdot \exp\left(-(4\pi \cdot R\sigma/\lambda)^2\right) \qquad \eta_{mb}(\lambda) = 1.2 \cdot \eta_a(\lambda) \qquad \eta_F(\lambda) = \exp\left(-(0.33\text{mm}/\lambda)^2\right)$$

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[2] F=f/D is the focal ratio of the system. A radius expressed in units of Fλ does not depend on the optical system: 1.03Fλ is the FWHM (or HPBW), 1.22Fλ is the radius of the 1st dark ring for a diffraction through a circular hole.
[3] See Gereve et al, A&A 133 271-284 1998, and Baars 2007 "The Parabolical Reflector Antenna in Radio Astronomy and Communication". Greve and Baars formulations are incompatible regarding energy normalization, but they both use the Ruze law (gaussian beam approximation) for the efficiencies. My method is based on Baars equations with a "gaussian tapered main beam" as eq. 6.37 in Goldsmith "Quasioptical Systems".

For comparison figure 2 shows the efficiencies obtained with the Ruze law and relative powers.
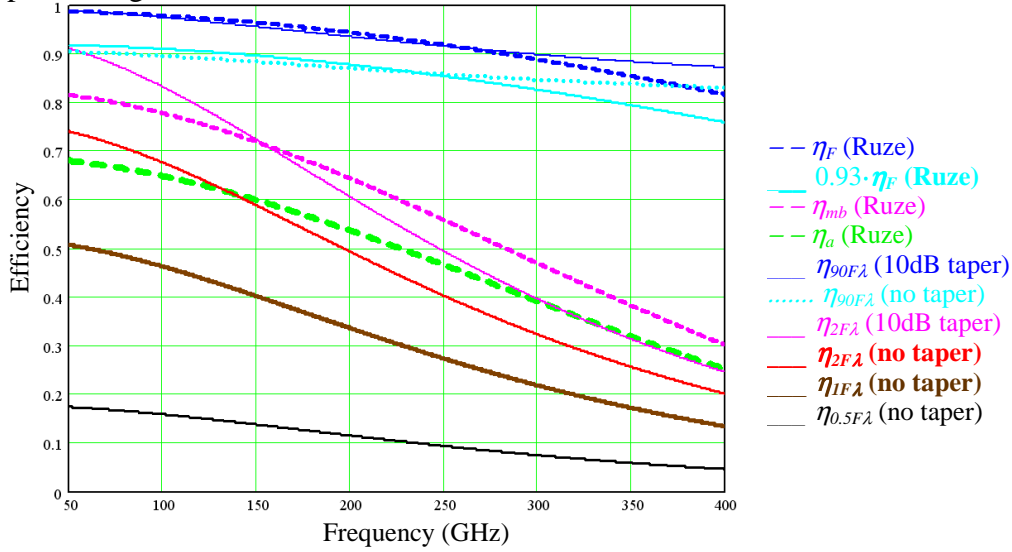

Figure 2. Efficiencies and relative powers at the 30m telescope. Blue dashed line = forward efficiency fit to the 30m website values. Green and magenta dashed lines = aperture and main beam efficiencies using the Ruze law. Blue and magenta solid lines = relative powers at 90Fλ (~ forward plane) and 2Fλ of a 10dB edge taper diffraction pattern built with ATT. Cyan dotted line, red, brown and black solid lines = relative powers in squares (bare pixels) of sizes 90Fλ, 2Fλ, 1Fλ, and 0.5Fλ. Cyan solid line = 93% of the forward efficiencies from the 30m website.

Though my version of ATT doesn't fit well the Ruze law (differences up to 10%) and needs more work, it has the advantage to include the taper function as a variable. For instance it shows that the relative power at 2Fλ is 20% higher for a 10dB edge taper than for a bare pixels. In addition it allows calculating the relative power at any radius of the diffraction pattern. For instance, GISMO pixels cut the diffraction pattern at 1Fλ, not at the 2Fλ main beam.

My 10dB edge taper forward efficiency is close to the values from the 30m web site, and since the relative power at big radius is very time consuming I use 93% of the 10dB fit for the "no taper" forward efficiency; at GISMO central frequency the result is close to Dominic's value, however **I would appreciate clarifications about the meaning of Dominic's forward efficiency "fiducial" value (I bet this is not a scaling from 10dB taper to bare pixel).**

Table 2. Summary of Dominic's attenuation factors versus mine:

$\varepsilon_D / \varepsilon_S$	t_D / t_S	$A\Omega_D / A\Omega_S$	$\eta_{sysD} / \eta_{sysS}$	η_{MBD} / η_{MBS}	ξ_D / ξ_S
0.2/0.08 = 2.5	0.8/0.9 = 0.9	3.9/2.5 = 1.6	0.35/0.52 = 0.7	0.68/0.60 = 1.1	0.88/0.90 = 1.0

We have to understand the discrepancies and reach a consensus for the values.

3) Background power

The occupation number gives the average number of photons in one of the quantum states available in the detection chain. Dominic's "efficiencies term" ($\eta_{sys}[(1-\xi) + \xi\varepsilon]$) is the standard procedure used for counting the photons: "atmosphere in the forward efficiency, other contributors up to the instrument in the spillover, and a receiver term for the instrument itself"[4]; in terms of brightness temperatures the total flux is $T_{sys} = (T_{atm}+T_{spill}) + T_{rec} = T_{sky} + T_{rec}$.

Figure 3 shows curves of the occupation numbers obtained using either Dominic's formula and attenuation factors ($n_D(\nu)$), or his formula with my attenuation factors ($n_{Ds}(\nu)$), or my formula including the telescope optics ($n_S(\nu)$). The 1st graph has no receiver contribution, the 2nd one includes the permanent filters, and the 3rd one includes the neutral density filter.

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[4] Private discussion with Roberto Neri, and T_{sys} equation follows Downes's "Radio Astronomy Techniques". For the spillover the standard assumption is $\varepsilon_{ground} = 1$, but some studies show it can be as low as 0.3 (ref lost).

The occupation number equations used for figure 3 and the calculations of detected power are:

$$n_D(\nu) = \eta_{sysD}\,[1-\xi_D + \xi_D\,\varepsilon]\;n_\nu(T_a)\,(t_{nd}) \;+\; (n_{Dfilters}(\nu,t_{nd}))$$

$$n_{Ds}(\nu) = \eta_{sysS}\,[1-\xi_S + \xi_S\,\varepsilon]\;n_\nu(T_a)\,(t_{nd})$$

$$n_S(\nu) = n_{Ds}(\nu) + \xi_S\,[1-t_t]\,\eta_o\cdot t_f^{\;nfilters}\cdot t_l\;n_\nu(T_t)\,(t_{nd}) \;+\; (n_{Sfilters}(\nu,t_{nd}))$$

$$n_\nu(T) = \cfrac{1}{\exp\!\left(\cfrac{h\nu}{kT}\right)-1}$$

Filters: $n_{f300}(\nu)=\xi[1-t_f^{\,nt}]\eta_o\cdot t_f^{\,nN+He+c}\cdot t_l(t_{nd})n_\nu(300K)$ $n_{f77}(\nu)=\xi[1-t_f^{\,nN}]\,t_f^{\,nHe+c}\cdot t_l(t_{nd})n_\nu(77K)$ $n_{f4}(\nu)=\xi_S[1-t_f^{\,nHe}\cdot t_l(t_{nd})]\,t_f^{\,nc}\,n_\nu(4K)$
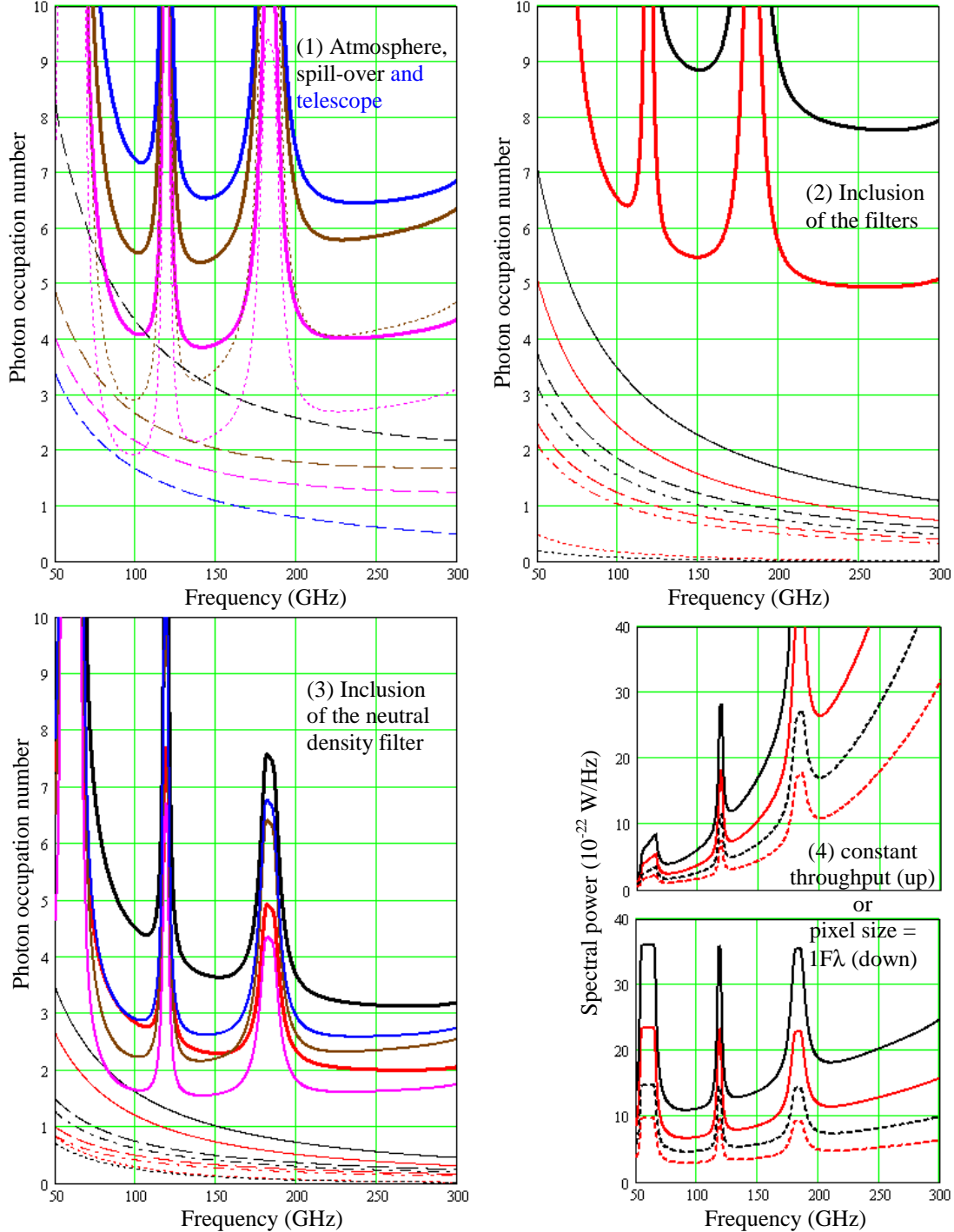

Figure 2. Occupation numbers and spectral power for observing conditions similar to Dominic ($w = 5\,mm$, $\theta = 45°$).
(1) Solid lines: magenta = n_D, brown = n_{Ds}, blue = n_S; dots = atmosphere; dash: magenta & brown = spillover; blue = telescope, black = spill-over + telescope. (2) red = $n_{Dfilters}$, black = $n_{Sfilters}$, dash = 300K, "dadot" = 77K, dot = 4K, thin solid = sum of filters, thick solid = addition to n_D and n_S. (3) Inclusion of t_{nd}, same colors as in (1) and (2). (4) Illustration of the effect of the pixel size function (see spectral power P_ν in the text); up: $A\Omega_D$, down: $A\Omega =(\pi/4)(c/\nu)^2$ (1Fλ pixel). red = Dominic factors, black = my factors, dash = with neutral density filter, solid = without it.

As expected, I find the same curves as Dominic when I use his factors and equation. But my attenuation factors, the inclusion of the telescope and the filters, gives different curves.

- **The big difference between Dominic and my spillover is mainly due to the difference between our forward efficiencies!**

- With 1% loss per mirror, the telescope optics doesn't appear in the standard description, but as Figure 3 shows, its contribution may be very important. **Where are photons produces by the telescope in this procedure?** Maybe in ξ_D since my telescope + ground ≈ Dominic's spillover, but this is in contradiction with counting the telescope into η_{sysD}!

- Dominic's equation and graphs of the occupation number include the atmosphere and spillover, but not the receiver (nevertheless counted in the calculation of the detected power). **Why Dominic does not count the photons from the filters into the occupation number?**

- The curves show that the 4K stage is negligible only without neutral density filter.

The product of occupation number n times number of states available in the system $2A\Omega/\lambda^2$ (for 2 polarizations) times energy of a photon $h\nu$ gives the energy detected in a frequency band[5] $d\nu$. The integral of this spectral power P_ν over the instrument bandwidth $\Delta\nu$ gives the power detected P, which can be approximated with a simple formula using the central frequency ν_c:

$$P_\nu(\nu) = \frac{2A\Omega \cdot \nu^2}{c^2} \cdot h\nu \cdot n(\nu) \quad [W/Hz] \qquad P = \int_{\Delta\nu} P_\nu(\nu)\,d\nu \; [W] \approx P_\nu(\nu_c)\Delta\nu \; [W]$$

The approximation $P_\nu(\nu_c)\Delta\nu$ is valid when P_ν is roughly linear in $\Delta\nu$, **so Domininc's formula is not valid when the opacity peaks of the atmosphere are in the bandwidth.**

The detected power is not a natural function of the frequency. So a plot of power function of frequency implies that the instrument central frequency is the variable, and in the integration of the spectral power, $\Delta\nu$ and $A\Omega$ may also be variables. For example: $\Delta\nu$ = constant or $\sim\nu$ (constant $\Delta\nu/\nu$) or band-passes suited to each atmospheric window, and $A\Omega$ = constant or $\sim 1/\nu^2$ (constant relative pixel size in Fλ) (see Figure 3, graph (4)). As shown in figure 4, Dominic's curves uses the approximation formula with a constant relative bandwidth $\Delta\nu/\nu$ =15%, a constant throughput $A\Omega$, and a constant 5pW contribution from the cryostat stages. **Dominic's power curves are ill defined**, but this is not really problematic since the relevant information is the calculation of the power passing through GISMO band-pass filter.
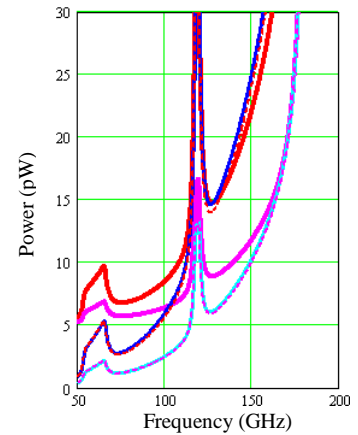

Figure 4. "Frequency gliding" power. Red = Dominic's factors with $\Delta\nu/\nu$=15%, $A\Omega_D$, and P_{rec}=5pW (solid) or $P_{rec}=\Sigma_{filters}P(\nu)$ (dash). Blue = my factors with $\Delta\nu/\nu$=15%, $A\Omega_S$, and $P_{rec}=\Sigma_{filters}P(\nu)$. Magenta & Cyan = Dominic & me with t_{nd}.

Table 3. Detected powers for Dominic's factors and mine, and various observing conditions:

Observing conditions	[w=1mm,θ=75°] ; [w=5mm,θ=45°] ; [w=7mm;θ=25°]	[not relevant]		
Neutral density filter	With		Without	
Attenuation factors	Dominic	Samuel	Dominic	Samuel
P_{atm} (pW)	1 ; 4 ; 8	1 ; 4 ; 8	2 ; 10 ; 20	2 ; 10 ; 20
P_{spill} (pW)	3	2	7	6
P_{tel} (pW)	-	1	-	3
P_{300} (pW)	1	1	4	3
P_{77} (pW)	1	1	3	3
P_4 (pW)	1	0	0	0
P_{tot} (pW)	7 ; 10 ; 14	7 ; 10 ; 14	16 ; 24 ; 34	17 ; 25 ; 35

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[5] One can recognize Planck's law of the blackbody brightness in frequency domain, times detection efficiencies.

Using Dominic's factors I find the same values as him for the sky contribution, but not for the cryostat (in his document P_{300} = 1pW, P_{77} = 1pW, P_4 = 3pW). **I don't understand Dominic's value for the 4K stage, and the fact that his instrument power is the same 5pW at all the frequencies, whether the neutral density filter is used or not**. The results from the table may be misleading, indeed the **apparent compatible results are only due to a fortuitous compensation of incompatible throughputs and system efficiencies**: $A\Omega_D \cdot \eta_{sysD} \approx A\Omega_S \cdot \eta_{sysS}$. In addition the table shows results calculated with three observing conditions ("good", "Dominic's typical", "bad") defined identically for Dominic's factors and for mine, but **considering that Dominic's observing conditions are pessimistic**, the typical values for the background power should be closer to the "good" case, which makes a **significant difference when the neutral density filter is used**.

- The mean number of modes available in the system is: $2A\Omega_D/\lambda_G^2 = 2$ $2A\Omega_S/\lambda_G^2 = 1.4$

4) Noise Equivalent Power

The fluctuations of the number of photons in a given state are $\sqrt{\overline{\delta n^2}} = \sqrt{\overline{n} + \overline{n}^2}$; the 1st term is the "shot noise" (thermodynamic, poissonian), and the 2nd term is the "radiometric noise" or "bunching noise" (interferences between bosons). The fluctuations of energy absorbed are: $\sqrt{\overline{\delta W^2}} = h\nu \cdot \sqrt{\overline{\delta n^2}}$. The bunching is proportional to the space and time coherences of the photons: $(\Delta_s \cdot \Delta_t) = 1/g$, where g is the number of states (or modes) illuminated for each polarization the system. With $2g\overline{n}$ photons in the available cells of the phase space, and an integration time t, the power can be written $P = h\nu \cdot 2g\overline{n}/t$. With $\Delta_t = \Delta\nu \cdot t$, and an integrator bandwidth $B=1/2t$, the Noise Equivalent Power[6] is:

$$NEP = \sqrt{\frac{2}{t} \cdot \overline{\delta W^2}} = h\nu \sqrt{\frac{2}{t} \cdot 2g\left(\overline{n} + \overline{n}^2\right)} = \sqrt{2h\nu \cdot P + \frac{\overset{\Delta}{s}}{\Delta\nu} \cdot P^2} = \sqrt{NEP_p^2 + NEP_b^2} \quad [W/\sqrt{Hz}]$$

The difference with Dominic's equation is that his NEP_{occ} applies to one mode only. Using Δ_s in the bunching term rather than $\lambda^2/A\Omega$ (inverse number of modes per polarization) allows to consider that all the modes landing on the detector are not illuminated equally. Systems producing special beams, like antennas, feedhorns or other gaussian optics, illuminate few modes in general; for example in a monomode horn $1/\Delta_s = 1$, independently of the physical size of its aperture. For optical systems absorbing all the photons coming through a given aperture with a given efficiency, like GISMO, the number of illuminated modes per polarization may be estimated with $A\Omega/\lambda^2$, but only if the throughput is bigger than the coherence since there can't be less than one mode illuminated. Without going through the rigors of quantum mechanics[7], a semiclassical approach[8] allows to calculate the spatial coherence factor Δ_s as the normalized covariance of the fluctuation of the intensity I in the system; for Lambertian sources illuminating a uniform efficiency detector of size r:

$$\overset{\Delta}{s}(r) = \frac{1}{(r)^4} \iint_{r^2} \iint_{r^2} I(x - x', y - y') \cdot dxdy \cdot dx'dy'$$
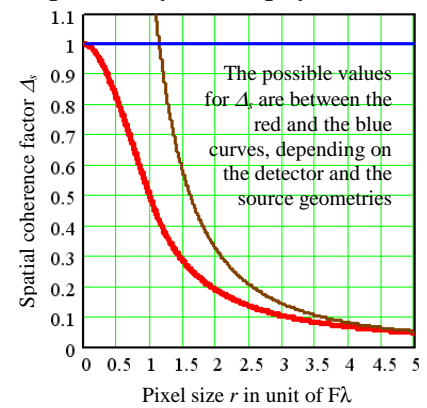
The photons are spread at least over a diffraction figure,

Figure 5. Δ_s and its asymptotes. Red = $\Delta_s(r)$ from the diffraction pattern, blue = 1 mode, brown = $\lambda^2 / A\Omega$ = $4/(\pi r^2)$. Possible values: red < Δ_s < blue. For GISMO $r =1$.

Plot axis labels: Spatial coherence factor Δ_s (y-axis, 0 to 1.1); Pixel size r in unit of Fλ (x-axis, 0 to 5). Text in plot: "The possible values for Δ_s are between the red and the blue curves, depending on the detector and the source geometries"

[6] "*Equivalent radiant power producing a signal-to-noise ratio of unity at a detector output for a given modulation frequency, operating wavelength, and effective noise bandwidth*" Federal Standard 1037C, telecom glossary 2000.
[7] J.Zmuidzinas, Applied Optics, Vol. 42, No 25, Sep. 2003.
[8] J.M.Lamarre, Applied optics, Vol 25, No. 6, 1986.

giving the minimum Δ_s, and at most uniformly[9], giving the maximum Δ_s. Figure 5 shows the possible values for the spatial coherence factor and its asymptotes: $\Delta_s = 1$ for coherent beams, and $\Delta_s = \lambda^2/A\Omega$ for incoherent beams.

For the background is extended so $\Delta_s = 1$, however the literature can be interpreted such that $\Delta_s(1F\lambda) = 0.5$ (red curve in figure 5) for multimode detectors independently of the source. **According to Dominic $A\Omega_D/\lambda_G^2 = 1$, so with $\Delta_s = 1$ one gets $NEP_D = \sqrt{2}\ NEP_{occD}$, but in his document Dominic uses NEP_{occ} as the detected NEP, so if I'm not wrong this means he underestimates the actual noise in the detector.**

With my factors the relation between NEP_{occ} and detected NEP is more complicated since $A\Omega/\lambda^2 \neq 1$ implies different ratios for shot noise and bunching noise.

The bunching is not poissonian, so with several components i:

$$NEP_{TOT} \neq \sqrt{\sum_i NEP_i^2} \quad \text{but} \quad NEP_{TOT} = \sqrt{NEP_{p,TOT}^2 + NEP_{b,TOT}^2}$$

$$\text{with} \quad NEP_{p,TOT} = \sqrt{\sum_i NEP_{p,i}^2} \quad \text{and} \quad NEP_{b,TOT} = \sum_i NEP_{b,i}$$

Table 4. NEPs for a GISMO pixel ($2A\Omega/\lambda^2$ modes), using Dominic's factors and mine with $\Delta_s = 1$, the values in small font between brackets details the shot and bunching components, the total NEP using $\Delta_s = 0.5$ is also given because of the vagueness of the literature:

Observing conditions	[w=1mm,θ=75°] ; [w=5mm,θ=45°] ; [w=7mm;θ=25°]	[not relevant]		
Neutral density filter	With		Without	
Attenuation factors	Dominic	Samuel	Dominic	Samuel
NEP_{atm} (10^{-17} W/$\sqrt{Hz}$)	1 [1,1] ; 4 [3,3] ; 7 [4,5]	1 [1,1] ; 4 [3,3] ; 7 [4,5]	2 [2,1] ; 8 [4,7] ; 15 [6,14]	2 [2,1] ; 8 [4,7] ; 16 [6,13]
NEP_{spill} (10^{-17} W/$\sqrt{Hz}$)	3 [2,2]	3 [2,2]	6 [4,5]	5 [3,4]
NEP_{tel} (10^{-17} W/$\sqrt{Hz}$)	-	2 [2,1]	-	3 [2,2]
NEP_{300} (10^{-17} W/$\sqrt{Hz}$)	2 [2,1]	2 [2,1]	4 [3,2]	4 [3,2]
NEP_{77} (10^{-17} W/$\sqrt{Hz}$)	2 [2,1]	2 [2,1]	3 [2,2]	3 [2,2]
NEP_4 (10^{-17} W/$\sqrt{Hz}$)	1 [1,0]	1 [1,0]	1 [1,0]	0 [0,0]
NEP_{tot} (10^{-17} W/$\sqrt{Hz}$)	6 [4,5] ; 8 [4,7] ; 11 [5,9]	6 [4,5] ; 8 [5,7] ; 11 [5,10]	12 [6,11] ; 17 [7,16] ; 24 [8,23]	13 [6,11] ; 18 [7,17] ; 25 [8,23]
$NEP_{tot}(\Delta_s=0.5)$	5 [4,3] ; 7 [4,5] ; 9 [5,7]	5 [4,3] ; 7 [5,5] ; 9 [5,7]	9 [6,8] ; 13 [7,11] ; 18 [8,16]	10 [6,8] ; 14 [7,12] ; 19 [8,17]

The NEPs obtained with Dominic's factors are similar to mine because the powers are similar (fortuitous compensation of incompatible throughputs and system efficiencies).

As expected from the relation between NEP_{occD} and NEP_D, I find results $\sqrt{2}$ bigger than Dominic for the atmospheric NEP including spillover. For the cryostat, this factor and the powers discrepancy already mentioned explain the difference with his "*in-band photon noise from the instrument*": $NEP_{inst} = 3 \cdot 10^{-17}$ W/$\sqrt{Hz}$.

Dominic gives $NEP_{det} = 4 \cdot 10^{-17}$ W/$\sqrt{Hz}$ for the detector intrinsic noise, which means GISMO should be background limited for all observing conditions.

Remark: Using $\Delta\nu/\nu = 15\%$ I recover the same curves as Dominic for the "frequency gliding" NEP_{occ}. Obviously with $A\Omega$ in the equation "frequency gliding $NEPs$" give different curves.

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[9] This paragraph is my interpretation of Lamarre and Zmuidzinas, but it implies multimode detectors have more modes illuminated for a point source than for an extended source. This seems counter intuitive and in contradiction with Lamarre's Fig. 1. that I recover using the diffraction pattern (see Figure 5.).

5) Sensitivities

The noise equivalent flux density (NEFD) is well adapted to describe point source sensitivity (flux density ~ power), whereas the noise equivalent temperature (NET) is well adapted to describe extended source sensitivity (flux per solid angle ~ temperature).

In both cases astronomers like quantities equivalent to sources out of atmosphere, independent of the pixels geometry (HPBW gives a natural standard size) and directly proportional to the integration time for given observing modes, hence the practical definitions:

$$NEFD = \frac{\eta_{obs}}{\sqrt{2}} \frac{NEP_{1F\lambda} \cdot 1mJy}{P_{1F\lambda,1mJy}} \quad [Jy \cdot \sqrt{s}] \qquad\qquad NET = \frac{\eta_{obs}}{\sqrt{2}} \frac{NEP_{1F\lambda} \cdot 1K_{RJ}}{P_{1F\lambda,1K_{RJ}}} \quad [K \cdot \sqrt{s}]$$

The factor $\sqrt{2}$ is due to the conversion from bandwidth to time: for a quantity X (Power, Flux, Temperature), the noise equivalent is in unit$(X)/\sqrt{Hz}$, meaning that for an system bandwidth B, a signal-to-noise = 1 is obtained when $X_{1\sigma}=NEX\cdot\sqrt{B}$. With $B=1/2t$, where t is the integration time, a practical formulation is NEX [unit$(X)\cdot\sqrt{s}$] = NEX [unit$(X)/\sqrt{Hz}$] $/\sqrt{2}$.

The observing efficiency η_{obs} has two components: a factor $\sqrt{2}$ due to the subtraction of the background in an image (signal - sky), and a modulation efficiency γ that can be regarded as an attenuation factor counting for the time effectively spent on-source or as a "spreading of instrumental response onto multiple pixels".

Replacing the observing mode efficiencies and source powers with their equations:

$$NEFD = \frac{\gamma \cdot NEP_{1F\lambda}}{A \cdot \Delta\nu \cdot \eta_{sys}(1-\varepsilon)\eta_{1F\lambda}} \quad [Jy \cdot \sqrt{s}] \qquad NET = \frac{\gamma \cdot NEP_{1F\lambda}}{A\Omega_{1F\lambda} \cdot \Delta\nu \cdot \eta_{sys}\xi \cdot 2k\nu^2/c^2} \quad [K \cdot \sqrt{s}]$$

Two striking differences with Dominic's formula for the NEFD:
> **(1) His bandwidth to time $\sqrt{2}$ factor is on the numerator rather than the denominator.**
> **(2) He uses the beam efficiency, so either his NEFD is defined for a full beam (his NEP must be calculated for 2Fλ, so 4 pixels), or he should rather use the 1Fλ efficiency.**

I define the parameter γ as an effective time factor depending on the observing mode chosen, but in Dominic's and Johannes documents this parameter is rather related to the number of pixels necessary to cover a point source (that their values are ~2 pixels to cover the full main diffraction beam up to the first dark ring in one dimension hence the value of the time-series γ, and ~4 pixels to cover the full main beam in two dimensions hence the map γ). But their documents are not clear, so **how did Dominic calculate his γ factors? Is their definition equivalent to the effective time factor I use for my calculations?** Assuming it is equivalent (necessary for the comparison of Dominic's method versus mine), one can compare the values of the modulation efficiency for Lissajou (γ_L) and linear source scan (γ_s) as given by Dominic, to the value in terms of effective integration time for On/Off (γ_{oo}) and direct integration (γ_d) assuming respectively 45% and 80% time on-source:

$$\gamma_L = 4.07 \qquad \gamma_s = 2.06 \qquad \gamma_{oo} = 1/\sqrt{0.45} = 1.49 \qquad \gamma_d = 1/\sqrt{0.8} = 1.12$$

To get rid of the background on the linear source scan a simple subtraction of a pixel off-source to the pixel on-source should be enough, isn't it? If that is the case then, the time on-source is half the total time of the two pixels, so the modulation efficiency should be the similar for time-series (γ_s) as for an observation flowing the source in On/Off (γ_{oo}), isn't it?

Using **Dominic's formula with his parameters** to calculate the map NEFD (γ_L) without neutral density filter for the atmosphere + spillover (using his NEP_{occ}), the instrument (with NEP_{inst}) and the detector (his NEP_{det}) **gives** respectively 40, 28 and 38 mJy·$\sqrt{s}$, about **1.8 times the values in his document!** Beside in both Dominic and my formulas the **NEFD is proportional to the modulation efficiency, but this is not the case of the results given by Dominic**. I couldn't reproduce Dominic's NEFD curves even with a 1.8 scaling on his formula.

Tables 5 and 6 display the results of my calculations of NEFD and NET for the 0.9Fλ GISMO pixel size (close enough to 1Fλ to use them as the GISMO standard sensitivities), using Dominic and my attenuation factors for the Lissajou and "following-source" observing modes. My sensitivities are not as good as Dominic's document, but the numerous differences between the formulas and results make any interpretation highly speculative. The priority should be to reach agreement on the formulas before arguing the meaning of the values.

Table 5. Map NEFD and NET (Dominic's Lissajou mapping factor γ_L)

Observing conditions	$[w=1\text{mm},\theta=75°]$; $[w=5\text{mm},\theta=45°]$; $[w=7\text{mm};\theta=25°]$ | [not relevant]			
Neutral density filter	With		Without	
Attenuation factors	Dominic	Samuel	Dominic	Samuel
$NEFD_{atm+spill}$ (mJy·$\sqrt{s}$)	28 ; 54 ; 106	14 ; 28 ; 56	23 ; 47 ; 96	11 ; 24 ; 51
$NEFD_{tel}$ (mJy·$\sqrt{s}$)	-	8 ; 9 ; 12	-	6 ; 7 ; 9
$NEFD_{cryostat}$ (mJy·$\sqrt{s}$)	25 ; 30 ; 40	13 ; 16 ; 21	18 ; 22 ; 29	10 ; 12 ; 15
$NEFD_{det}$ (mJy·$\sqrt{s}$)	31 ; 37 ; 49	17 ; 21 ; 28	12 ; 15 ; 20	7 ; 8 ; 11
$NEFD_{tot}$ (mJy·$\sqrt{s}$)	**54 ; 83 ; 141**	**31 ; 47 ; 80**	**39 ; 66 ; 120**	**23 ; 39 ; 69**
NET_{tot} (mK·$\sqrt{s}$)	2 ; 2 ; 3	2 ; 2 ; 3	3 ; 4 ; 5	3 ; 4 ; 6

Table 6. Direct integration NEFD and NET (my following source factor γ_d)

Observing conditions	$[w=1\text{mm},\theta=75°]$; $[w=5\text{mm},\theta=45°]$; $[w=7\text{mm};\theta=25°]$ | [not relevant]			
Neutral density filter	With		Without	
Attenuation factors	Dominic	Samuel	Dominic	Samuel
$NEFD_{atm+spill}$ (mJy·$\sqrt{s}$)	8 ; 15 ; 29	4 ; 8 ; 15	6 ; 13 ; 26	3 ; 7 ; 14
$NEFD_{tel}$ (mJy·$\sqrt{s}$)	-	2 ; 3 ; 3	-	2 ; 2 ; 2
$NEFD_{cryostat}$ (mJy·$\sqrt{s}$)	7 ; 8 ; 11	4 ; 4 ; 6	5 ; 6 ; 8	3 ; 3 ; 4
$NEFD_{det}$ (mJy·$\sqrt{s}$)	8 ; 10 ; 13	5 ; 6 ; 8	3 ; 4 ; 5	2 ; 2 ; 3
$NEFD_{tot}$ (mJy·$\sqrt{s}$)	**15 ; 23 ; 39**	**9 ; 13 ; 22**	**11 ; 18 ; 33**	**6 ; 11 ; 19**
NET_{tot} (mK·$\sqrt{s}$)	0.4 ; 0.5 ; 0.7	0.4 ; 0.6 ; 0.7	0.8 ; 1.1 ; 1.5	0.8 ; 1.1 ; 1.5

As expected the Lissajou observing mode doesn't seem favorable for observations of known point sources. However it may be very good for mapping, but maps may show inhomogeneous distribution of the integration time on the sky, so how is calculated the modulation efficiency in that case? There are chances it is not constant everywhere. This problem is out of the scoop of the present document, but it is evocated in more details in the reply to Johannes report.

Remark: If the spatial coherence factor Δ_s is closer to the "incoherent beam" asymptote than to the "coherent" one (used in the formulas), the NEFDs should be smaller.

Conclusion.

Dominic Benford and I calculated the expected background power and fluctuations on GISMO pixels for various atmospheric conditions at the Pico Veleta 30m telescope. Though the orders of magnitudes are comparable for identical conditions, there are a great number of discrepancies. Surprisingly they appear in all the steps of the calculation: definition of the typical atmospheric conditions, values of the various attenuation factors, estimations of the contribution of the cryostat stages, compatibility between the NEP and NEFD formulas. I recommend doing another iteration with the GISMO team to understand the discrepancies and hopefully reach an agreement.